

Comparative Study of Antenna Array Calibration Methods Based on Symmetrical Mutual Coupling of 4-Element Linear Array

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[Summary]

Antenna array calibration is indispensable for accurate direction-of-arrival (DOA) estimation, because mutual coupling and amplitude/phase error on antenna array cause a significant error in the MUSIC spectrum. Although an asymmetric calibration method which independently calculates all elements of the calibration matrix enables precise array calibration, a large number of reference data set is required to find many unknown variables. On the other hand, the 28-GHz frequency band is used for high-speed wireless communication on the 5th generation mobile communication system, and a demand for DOA estimation will increase on the SHF band. In this paper, two calibration models using symmetrical mutual coupling of 4-element uniform linear array were compared to reduce the number of reference data set. As a result of 28-GHz band experiment, the DOA estimation error and MUSIC spectrum were improved by an appropriate calibration model and were close to that of the asymmetric calibration method with many unknown variables. Copyright © 2023 IEICE

1 Introduction

By using an array antenna, it is possible to estimate the direction of arrival of radio waves without mechanical scanning, and high-resolution direction-of-arrival estimation methods such as the MUSIC algorithm¹⁾ are known well. In the typical MUSIC algorithm, it is assumed that each antenna can ideally receive the incoming waves, but in actual array antennas, there are various error factors listed below, so it is difficult to obtain the theoretical results. This hinders accurate direction-of-arrival estimation. The first error factor is mutual coupling between antenna elements. When one antenna receives an incoming wave, re-radiation occurs due to the current excited on the element, and another antenna receives this, resulting in mutual coupling between elements. The second error factor is amplitude and phase errors of each element. Even if antenna elements, feed lines, and receivers with the same specifications are arranged in an array, there will be non-uniformity in the actual device, resulting in errors in amplitude and phase characteristics between the elements. The third error factor is considered the position error of the antenna element. In this paper, we assume that the position errors can be ignored and discuss an array calibration method that calibrates the mutual coupling between elements and the amplitude and phase errors of each element. Furthermore, in the 5th generation mobile communication system (5G), the 28-GHz band is used in addition to the sub 6-GHz band to perform high-speed, large-capacity wireless

communication using a wide frequency band, and direction-of-arrival estimation in the SHF band is expected to be necessary. As the carrier frequency increases, the loss of high-frequency devices and transmission lines generally increases, and the carrier phase difference due to line length differences also increases, so the amplitude and phase errors of each element tend to increase, and array calibration becomes more important.

The method of determining a calibration matrix using a wave source with known direction of arrival is considered useful in practice, since it is possible to calibrate the receiver's amplitude and phase errors as well. When the number of antenna elements is N , mutual coupling between elements is represented by a complex matrix of $N \times N$, and amplitude and phase errors are represented by N complex numbers. If we do not assume array symmetry and assume that all elements of the calibration matrix are independent, the mutual coupling between elements and the amplitude and phase errors can be expressed together by an $N \times N$ calibration matrix. This asymmetrical calibration method using the $N \times N$ matrix is versatile and can be applied to various antenna configurations, and enables highly accurate calibration, but it requires many calibration data sets because of the large number of unknown variables. On the other hand, with an antenna configuration such as a uniformly spaced linear array, it is possible to reduce unknown variables in the mutual coupling matrix due to its symmetry. However, since the

amplitude and phase errors of each element cannot be included in the mutual coupling matrix, the unknown variables of the mutual coupling coefficient and the amplitude and phase errors take the form of a product, making it difficult to directly solve each unknown variable. Previously proposed methods^{2), 3)} solve the product of the mutual coupling coefficient and the amplitude/phase error as a new unknown variable. There were two problems in that the number of unknown variables increased and the calibration results varied depending on the method of calculating the mutual coupling coefficients and the amplitude and phase errors from the new unknown variables. To overcome these problems, iterative calculation methods^{4), 5)} have been proposed for determining mutual coupling coefficients and amplitude/phase errors. The latter method allows calculations without increasing the number of unknown variables, so it has the feature of being able to perform array calibration with a small number of calibration data sets.

In this paper, we focus on the symmetrical mutual coupling coefficient and its iterative calculation method for array calibration with a small calibration data set and conduct an array calibration experiment using a 28-GHz band four-element uniformly spaced linear array antenna to compare several calibration methods. Specifically, array calibrations of a four-element uniformly spaced linear array are performed using the calibration model described in References 4) and 5) and a calibration model in which the order of the mutual coupling coefficient and amplitude/phase error is reversed. As a calibration result, we evaluated the error in direction of arrival estimation using the MUSIC algorithm and the peak value of the MUSIC spectrum and compared the two calibration models. We also compared the results of the calibration using only the amplitude and phase errors of each element and the asymmetrical calibration using the $N \times N$ matrix.¹

2 Array Calibration Models

2.1 Symmetrical Calibration Model #1

The number of elements of the receiving antenna is N , the positions of the receiving antenna elements are $x_1, \dots, x_N$, the number of calibration data sets is M , the complex

amplitude of the i th calibration signal is s_i , the angle of arrival is θ_i , and similar to the References 4) and 5), the received signal vector $\mathbf{r}_i$ is expressed using the calibration model of equation (1).

$$\mathbf{r}_i = \mathbf{C}\mathbf{\Gamma}\mathbf{a}(\theta_i)s_i + \mathbf{n}_i \quad (1)$$

$$\begin{aligned} \mathbf{a}(\theta_i) &= [a_{i,1}, \dots, a_{i,N}]^T \\ &= \left[e^{j\frac{2\pi}{\lambda}x_1 \sin \theta_i}, \dots, e^{j\frac{2\pi}{\lambda}x_N \sin \theta_i} \right]^T \end{aligned} \quad (2)$$

$$\mathbf{\Gamma} = \text{diag} \{ \gamma_1, \dots, \gamma_N \} = \text{diag} \{ \alpha_1 e^{j\phi_1}, \dots, \alpha_N e^{j\phi_N} \} \quad (3)$$

$$\begin{aligned} \mathbf{n}_i &= [n_1^{(i)}, \dots, n_N^{(i)}]^T \\ (i &= 1, \dots, M) \end{aligned} \quad (4)$$

Where, $\mathbf{a}(\theta)$ is the ideal mode vector of the linear array, and $\mathbf{\Gamma}$ is the diagonal matrix representing the amplitude error $\alpha_1, \dots, \alpha_N$ and phase error $\phi_1, \dots, \phi_N$ of each element, $\mathbf{C}$ is an $N \times N$ complex matrix representing mutual coupling between elements, $\mathbf{n}_i$ is the noise vector at the i th reception. The mutual coupling coefficient $\mathbf{C}^{(3)}$ in equation (5) is assumed based on the symmetry of the four-element uniformly spaced linear array antenna.

$$\mathbf{C} = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_1 & c_4 & c_5 & c_2 \\ c_2 & c_5 & c_4 & c_1 \\ c_3 & c_2 & c_1 & c_0 \end{bmatrix} \quad (5)$$

The unknown variables are $\{c_0, \dots, c_5, \gamma_1, \dots, \gamma_4\}$, and if the coefficients are normalized to $c_0 = 1, \gamma_1 = 1$, there are 8 unknown variables to solve. To calculate these unknown variables, it is necessary to obtain the received signal $\mathbf{r}_i$ when a wave source with a known arrival angle θ_i is received. Equation (1) includes the product of the unknown variables $\mathbf{C}$ and $\mathbf{\Gamma}$ and is difficult to solve directly, so a method that calculates the product of the unknown variables has been reported³⁾. However, the number of unknown variable products to be solved is 15, which is the same number as a 4×4 matrix (with any one element as the reference) that does not assume symmetry, and the effect of reducing unknown variables due to symmetry cannot be obtained. Furthermore, when the symmetry is not perfect, such as in actual measurement data, there is a problem in that the calibration results vary depending on the method used to convert the 15 unknown variable products into the 8 unknown variables. Therefore, in

¹ The material in this paper was presented in part at the IEICE Technical Report 9), and all the figures of this paper are reused from 9) under the permission of the IEICE.

this paper, we calculate the 8 unknown variables using the iterative calculations shown in references 4) and 5) to obtain the mutual coupling coefficient $\mathbf{C}$ and the amplitude/phase error $\mathbf{\Gamma}$.

2.2 Symmetrical Calibration Model #2

Considering that the mutual coupling $\mathbf{C}$ between receiving elements occurs at the receiving antenna, and the amplitude and phase errors $\mathbf{\Gamma}$ of each receiving element are caused by the feed lines and receivers after the receiving antenna, the order of $\mathbf{C}$ and $\mathbf{\Gamma}$ changes, and the calibration model is expressed as

$$\mathbf{r}_i = \mathbf{\Gamma} \mathbf{C} \mathbf{a}(\theta_i) s_i + \mathbf{n}_i \quad (6)$$

Reference 6) presents a calibration model for the order of $\mathbf{\Gamma} \mathbf{C}$, but does not assume the symmetry of mutual coupling, whereas this paper assumes the symmetrical mutual coupling of equation (5) for a four-element uniformly spaced linear array antenna, as described in the previous section. The unknown variables are $\{\alpha_0, \dots, \alpha_5, \gamma_1, \dots, \gamma_4\}$, and if the coefficients are normalized to $\alpha_0 = 1, \gamma_1 = 1$, there are 8 unknown variables to solve. To calculate these unknown variables, it is necessary to obtain the received signal $\mathbf{r}_i$ when a wave source with a known arrival angle θ_i is received. As described in the previous section, equation (6) includes the product of the unknown variable $\mathbf{C}$ and $\mathbf{\Gamma}$, and it is difficult to solve it directly. Therefore, we calculated the 8 unknown variables using the iterative calculations shown in references 4) and 5), and obtained the mutual coupling coefficient $\mathbf{C}$ and the amplitude/phase error $\mathbf{\Gamma}$.

3 Experiment

Figure 1 shows the experimental setup.

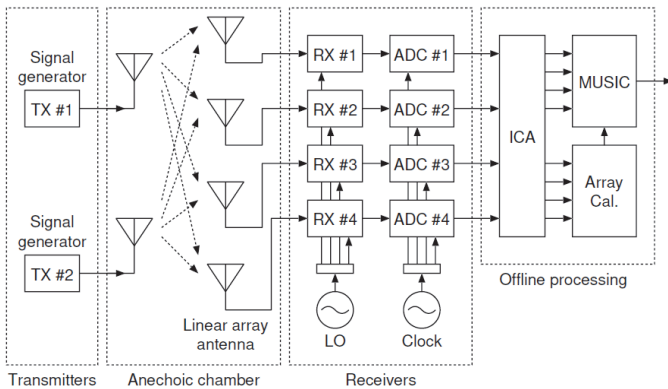


Figure 1 Experimental setup for array calibration

The transmitters (TX#1, #2) are general-purpose signal

generators to output OFDM signals with a carrier frequency of 28.25 GHz, a bandwidth of 100 MHz, a subcarrier spacing of 120 kHz, and QPSK modulation. The transmitting antenna is a horn antenna, and the receiving antenna is a four-element linear array with half-wavelength spacing using the sleeve antennas⁷⁾ formed on printed circuit boards. The distance between the transmitting and receiving antennas was 2.15 m, and the experiment was conducted in an anechoic chamber. The receiver consists of a 4-channel down converter (RX#1–#4) and a 4-channel A/D converter (ADC#1–#4) using a common local signal and a common A/D conversion clock signal for phase synchronization between channels. Although the signal wiring for each channel was designed to be as equal in length as possible, no individual phase correction was performed using variable phase shifters, etc., and phase differences due to cable length variations and individual differences in high-frequency devices remain. The A/D converters with simultaneous sampling capability of 4 channels digitize each IF signal from the down converter with a sampling frequency of 983.04 MHz, a resolution of 14 bits, and a time span of 4 ms, and then convert to I/Q signal with a sampling frequency of 245.76 MHz by the digital down converter (DDC). In the experiment, we received two signals with different directions of arrival, separated the signals using independent component analysis (ICA), calculated the covariance matrix, and performed array calibration. 25 data sets were obtained by setting the direction of arrival angle from -72° to $+72^\circ$ in 6° steps, and among them, 7 data sets of $\{-72^\circ, -48^\circ, -24^\circ, 0^\circ, +24^\circ, +48^\circ, +72^\circ\}$ were used for array calibration. The MUSIC spectrum after calibration was calculated by

$$P_{\text{MUSIC}}(\theta) = \frac{\tilde{\mathbf{a}}^H(\theta) \tilde{\mathbf{a}}(\theta)}{\tilde{\mathbf{a}}^H(\theta) \mathbf{E}_N \mathbf{E}_N^H \tilde{\mathbf{a}}(\theta)} \quad (7)$$

using the calibrated mode vector $\tilde{\mathbf{a}}(\theta) = \mathbf{C} \mathbf{a}(\theta)$ or $\tilde{\mathbf{a}}(\theta) = \mathbf{\Gamma} \mathbf{C} \mathbf{a}(\theta)$. Where, $\mathbf{E}_N$ is a matrix consisting of noise eigenvectors. To estimate the direction of arrival, we calculated the angle θ_i at which the MUSIC spectrum is maximum for each of the 25 data sets from -72° to $+72^\circ$, and calculated the peak value of the MUSIC spectrum $P_{\text{MUSIC}}(\theta_i)$ to evaluate the calibration accuracy.

4 Experimental results

Figure 2 shows the direction of arrival estimation error, and Figure 3 shows the peak value of the MUSIC spectrum.

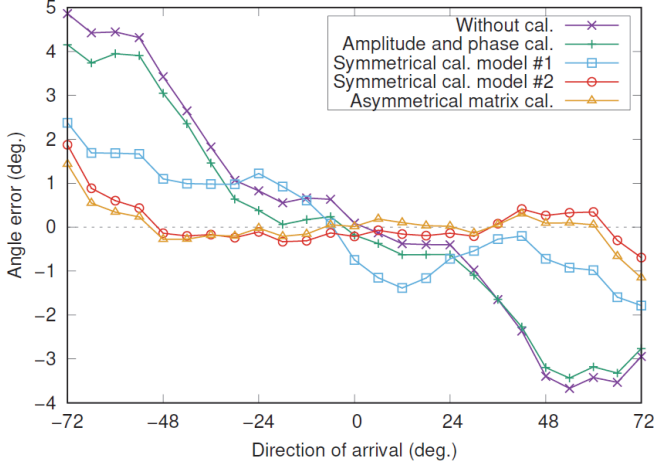


Figure 2 Direction of arrival estimation error

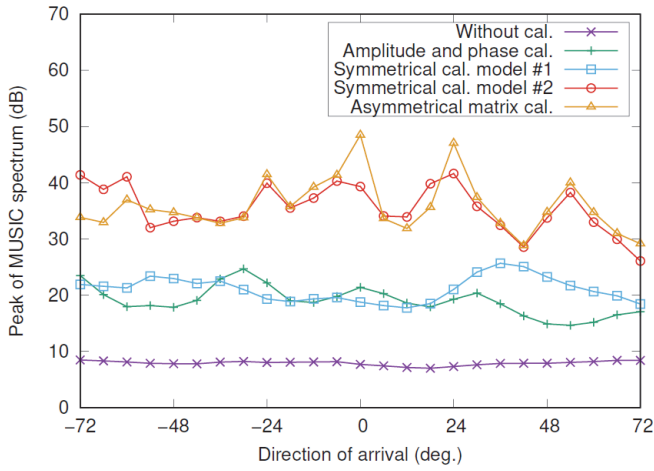


Figure 3 Peak value of MUSIC spectrum

“x” indicates no array calibration, “+” indicates calibration of only the amplitude and phase errors of each element, “□” indicates the symmetrical calibration model #1 ($\mathbf{CT}$), “○” indicates the symmetrical calibration model #2 ($\mathbf{\Gamma C}$), “△” indicates the asymmetrical calibration using 4×4 matrix. Although the peak value of the MUSIC spectrum is slightly improved by calibrating only the amplitude and phase errors of each element, the direction of arrival estimation error is not much different from the case without array calibration, and so the direction of arrival estimation error seems to be mainly due to mutual coupling between elements. The symmetrical calibration model #1 ($\mathbf{CT}$) shows an error in the direction of arrival over the entire angle, and the peak value of the MUSIC spectrum also hardly improves from the calibration of only the amplitude and phase errors. Therefore, the calibration model in which the mutual coupling exists after the amplitude and phase errors is considered to be inconsistent with the experiment. In the symmetrical calibration

model #2 ($\mathbf{\Gamma C}$), the direction of arrival estimation error decreases and the peak of the MUSIC spectrum increases, yielding results close to the asymmetrical calibration using a 4×4 matrix.

Figure 4 summarizes the relationship between the number of unknown variables and calibration accuracy. Figure 4(a) shows the Root-Mean-Square (RMS) value of the direction of arrival estimation error, and Figure 4(b) shows the average value of the MUSIC spectrum calculated using equation (8). Since the denominator of $P_{\text{MUSIC}}(\theta)$ is expressed as the sum of squares of the inner product of the mode vector and the noise eigenvector⁸⁾, the average value of the reciprocal number was used as the average value of the MUSIC spectrum.

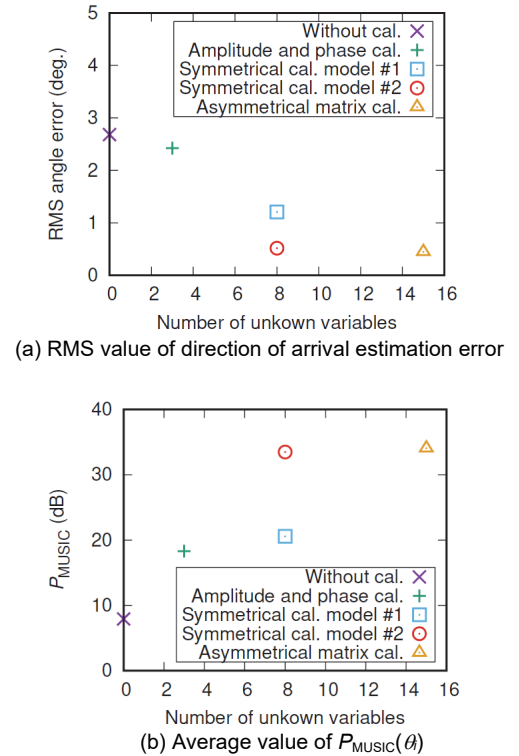


Figure 4 Comparison of array calibration results

$$P_{\text{MUSIC}} = \frac{1}{\frac{1}{L} \sum_{i=1}^L \frac{1}{P_{\text{MUSIC}}(\theta_i)}} \quad (8)$$

Where, L is the number of measurement points for the direction of arrival estimation, and in this experiment, $L = 25$. Figure 4 shows that the symmetrical calibration model #2 can achieve calibration accuracy close to the asymmetrical calibration by a relatively small number of unknown variables. It is considered that the symmetry of the mutual coupling coefficient shown in equation (5) and the calibration model shown in equation (6) are in good agreement with the experiment.

Table 1 shows the calibration values obtained using the symmetrical calibration model #2 (ΓC).

Table 1 Calibration value by calibration model #2

		Amplitude	Phase (deg.)
Amplitude and phase error	γ_1	1.00	0.0
	γ_2	0.81	-19.9
	γ_3	0.83	40.5
	γ_4	0.97	-15.9
Mutual coupling coefficient	c_0	1.00	0.0
	c_1	0.18	48.3
	c_2	0.09	-146.5
	c_3	0.06	28.6
	c_4	1.08	0.0
	c_5	0.19	46.0

The experimental results in Reference 3) showed a phase error of 21° (peak-to-peak) at a frequency of 2.4 GHz, whereas our experiment had a large phase error of 60° (peak-to-peak) due to the high frequency of 28 GHz, so it is thought that our experiment showed the large difference depending on the calibration models. Regarding the mutual coupling coefficient between elements, the diagonal components c_0 and c_4 have an amplitude of approximately 1, and the amplitude decreases in the order of the mutual couplings c_1 and c_5 between adjacent elements, the mutual coupling c_2 two elements apart, and the mutual coupling c_3 three elements apart. This trend is similar to the amplitude of the mutual coupling coefficient in reference 3) and is considered to be a valid result as a four-element linear array.

5 Conclusion

For the purpose of array calibration with a small calibration data set, we used an iterative calculation method assuming mutual coupling coefficients based on the symmetry of a four-element uniformly spaced linear array, and made comparison between the calibration model #1 which exists mutual coupling after amplitude and phase errors and the calibration model #2 which exists amplitude and phase errors after mutual coupling. As a result of experiments in the 28-GHz band, the calibration model #2 showed better characteristics in terms of direction of arrival estimation error and MUSIC spectrum. Therefore, the calibration model based on that mutual coupling occurs at the receiving antenna and

amplitude and phase errors occur at the feed lines and the receivers after the receiving antenna is considered more realistic. In addition, the calibration results using the calibration model #2 are close to the results of the asymmetrical calibration in which all elements of the 4×4 matrix are calculated independently, and so the assumption of mutual coupling coefficients based on symmetry is considered valid. From the above, it was found that highly accurate array calibration is possible with a relatively small number of calibration data sets using the symmetrical mutual coupling coefficients and the calibration model #2.

6 Acknowledgments

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