

# TECHNICAL NOTE

## - Random Error Function -

MP1590A

Network Performance Tester

ANRITSU CORPORATION

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## 1: Necessity of Random Error Insertion

Generally, error correction codes classified as block codes are used for correcting random errors (errors that can occur on actually circuits). The ITU-T G.709 Optical Transport Network (OTN) has adopted Reed-Solomon codes (RS (255, 239)) as a technique for Forward Error Correction (FEC). Because Reed-Solomon codes are block codes, pseudo-random errors must be generated to evaluate the performance of an FEC decoder, for example, by comparing the performance of FEC as against the theoretical value curve. The theoretical value curve shown in Figure 1 has been calculated under the condition of generating random errors.

When inserting errors at a specific error rate, however, the performance of FEC cannot be evaluated properly if errors are generated at even intervals. To correctly evaluate the performance of FEC, errors must be generated at random to create various states where errors can be corrected or where errors cannot be corrected.

For random errors, the error rate is satisfied over a long period of time. Within a short period of time, the error rate fluctuates as against the set value. This is close to the actually circuits and is therefore suitable for evaluating the performance of FEC.

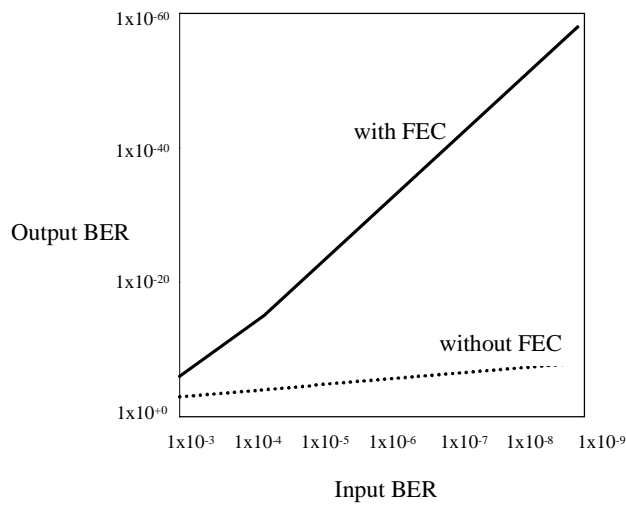


Figure 1 Output-Input BER

### When the error rate is 1/n

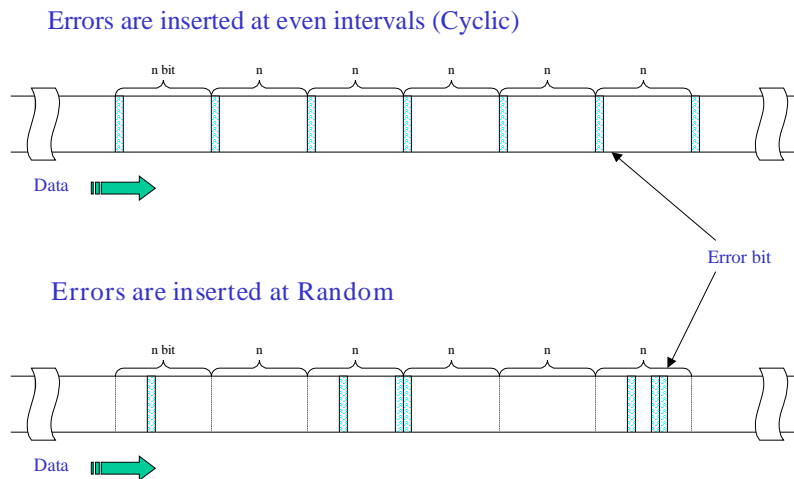


Figure 2 Error insertion

## 2: Principle of Random Error Insertion

### 2-1 Binominal Distribution and Poisson Distribution

When two types of events (A and B) can occur, trials are repeated and each trial is independent (the results of former trials do not affect the result of the current trial), and probability  $p$  of obtaining result A is constant throughout the trials, these trials are called Bernoulli Trials. For example, using the pieces in the game Othello, consider the probability of the white side of the pieces coming up when they are thrown repeatedly. If none of the pieces have any deformation and are coated with white and black paints of equal weight, this results in Bernoulli Trials with the probability of obtaining the white side  $p = 1/2$ .

The probability of obtaining result A  $k$  times when trials are repeated  $n$  times can be expressed by formula (1) shown below. This probability distribution is called a binominal distribution. In formula (1),  $\mu$  indicates the mean value and  $\sigma^2$  indicates the variance.

$$f(n, k) = \binom{n}{k} p^k (1-p)^{n-k} \quad \begin{cases} \mu = np \\ \sigma^2 = np(1-p) \end{cases} \quad \text{---(1)}$$

Explaining formula (1) by taking error generation as an example, it indicates the probability distribution for  $k$  bit errors in  $n$  bits. However, the calculation of formula (1) becomes more and more complicated as the value of  $n$  increases. It is actually difficult to directly calculate this formula with values assigned.

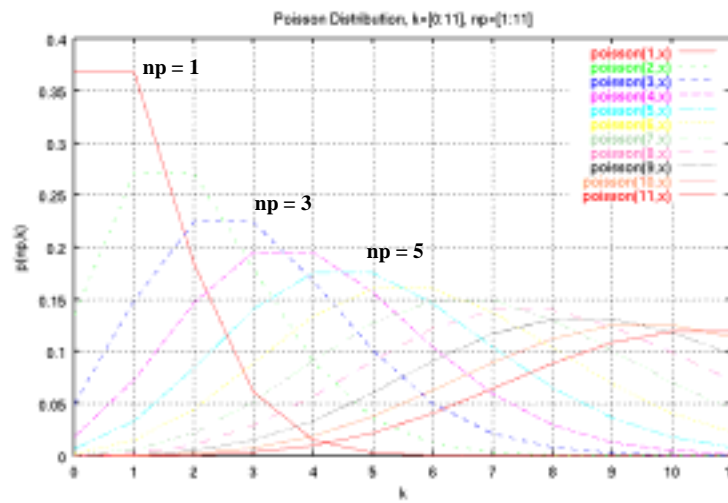
Now, assume  $\lambda = np$ , where  $\lambda$  is a fixed value. This indicates that  $p$  decreases when  $n$  increases and  $p$  increases when  $n$  decreases. Then, assume an extreme binominal distribution in which  $n \rightarrow \infty$  and  $p \rightarrow 0$ . This leads to the following formula:

$$f(k; \lambda) = e^{-\lambda} \frac{\lambda^k}{k!} \quad \begin{cases} \mu = \lambda \\ \sigma^2 = \lambda \end{cases} \quad \text{---(2)}$$

The probability distribution expressed in formula (2) is called a Poisson distribution. Since  $\lambda$  is fixed, this distribution is indicated as a distribution by variable  $k$ . To relate the Poisson distribution to the binominal distribution, assign  $np$  to  $\lambda$ . Then, formula (3) is obtained.

$$f(k; np) = e^{-(np)} \frac{(np)^k}{k!} \quad \text{---(3)}$$

Probability distribution of variable “k” when “np” is fixed (Poisson distribution)



**Figure 3 Poisson distribution**

Figure 3 shows the Poisson distribution as a graph. This graph indicates the probability distribution for k occurrences of an event as against mean value np.

For example, consider the probability of a baseball player with a batting average of .300 scoring k hits within 10 at-bats. In this case,  $\lambda = np = 10 \times 0.3 = 3$ ; so the player scores three hits on average. Actually, however, the player might score eight hits within 10 at-bats or only one hit within another 10 at-bats. Under these conditions, the probability that this player scores five hits within 10 at-bats is calculated using formula (3), as follows:

$$f(k = 5; np = 3) = e^{-(3)} \frac{(3)^5}{5!} = 0.1008188..... \quad \text{---(4)}$$

That is, the player scores five hits within 10 at-bats with a probability of approximately 10%.

This result is indicated by the probability distribution shown as the Poisson (3,x) graph in Figure 2. In the Poisson (3,x) graph, the peak comes at k = 3 because this player scores three hits on average. The probability that the player scores three hits within 10 at-bats is 22.4%. The probability goes down both when the number of hits increases and when it decreases.

### 2-2 Poisson Distribution and Error Rate

The following describes random error generation in relation to Poisson distribution.

In this example, p indicates an element error rate (error rate). np indicates the average error bit count  $\mu$  when the frame length is n bits. For example, when p is fixed,  $n = \mu / p$ .

Figure 4 indicates the result obtained by applying this error rate example to Figure 3. The frame length n increases as  $1 \times 10^3$ ,  $2 \times 10^3$ ,  $3 \times 10^3$ , .....bits when the error rate p is  $1 \times 10^{-3}$  and fixed, and the average error bit count increases as 1, 2, 3, and so on.

Probability distribution of variable “k” when “np” is fixed (Poisson distribution)

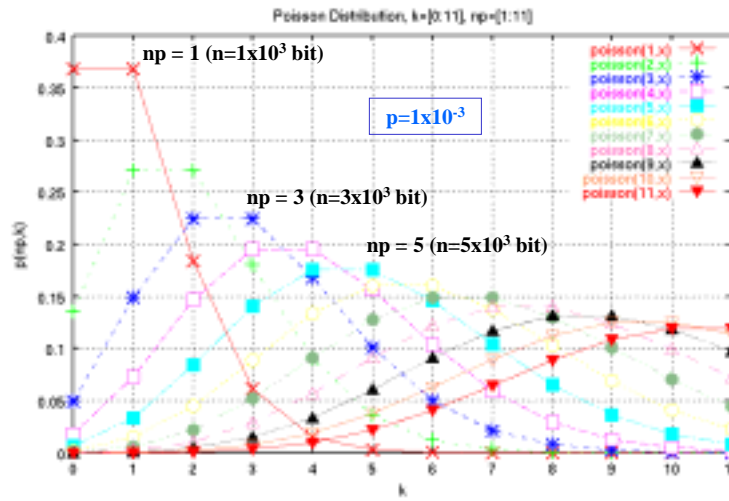


Figure 4 Distribution when  $p = 1 \times 10^{-3}$

In the figure 4, k indicates the error bit count. When the average error bit count np is 1 (Poisson (1,X)), the probability that one bit error ( $k = 1$ ) is the highest.  $k = 2$  indicates the probability that two out of  $1 \times 10^3$  bits error occur, and this probability is lower. In the same way, when  $np = 5$  (Poisson (5,x)), the probability that five bits error occur is the highest. The probabilities that more than five or less than five out of  $5 \times 10^3$  bits error occur are lower.

Random errors can be generated based on this probability distribution.

The following describes the relationship between error rates and Poisson distribution from another point of view. Figure 5 shows the Poisson distribution when error bit count k is fixed and average error bit count np is changed.

Probability distribution of variable “np” when “k” is fixed (Poisson distribution)

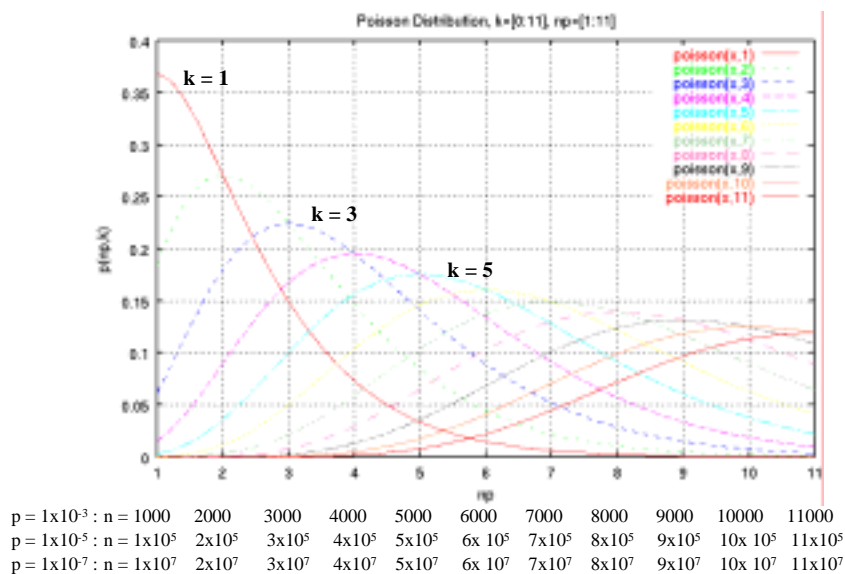
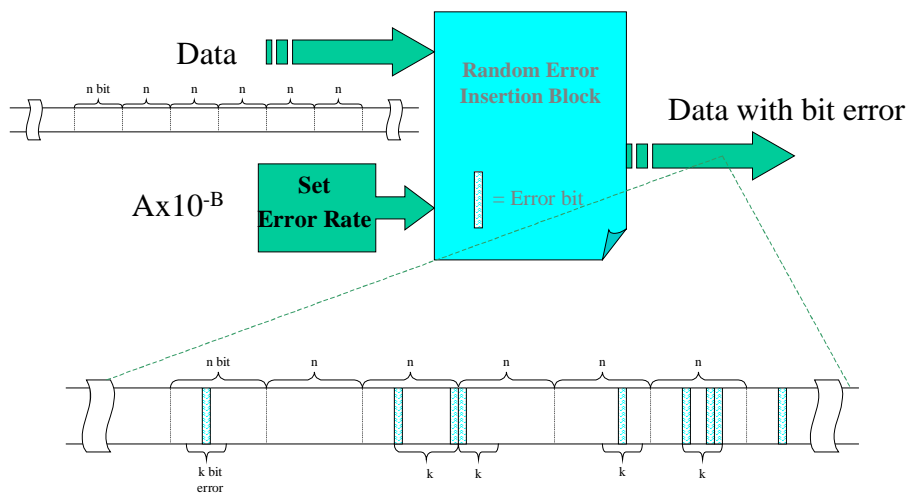


Figure 5 Poisson distribution considered by assuming np as a variable

Figure 5 (Poisson (x,1)) shows the probability distribution for one bit error ( $k = 1$ ) for np. When the

error rate is fixed at  $p = 1 \times 10^{-3}$  (on average, one out of 1000 bits), the peak comes at  $n = 1000$  (bits). When  $np = 2$ , the probability is that two out of 2000 bits error occur. This probability is lower than that for  $np = 1$ . In the same way, Poisson ( $x,3$ ) indicates the probability distribution for three ( $k = 3$ ) error bits. When the error rate is fixed at  $p = 1 \times 10^{-3}$ , the peak comes at  $n = 3000$  (bits).

Figure 6 shows an actual image of random error insertion.



*When data is separated into n-bit blocks, error bit count “k” inserted into each block has a Poisson distribution.*

**Figure 6 Image of random error insertion**

When data is separated into n-bit blocks, the error bit count k inserted into each block has a Poisson distribution. The value of the average error bit count  $np$  determines which of the graphs shown in Figure 4 indicates the distribution of error bit count k.

### 2-3 Specification for Random Error Insertion for the MP1590A

The random error insertion function of the MP1590A has been designed taking into consideration the randomization quality (each event occurs independently unaffected by previous or later events) and uniformity (each event occurs at the same probability). The MP1590A has satisfied these conditions to enable random error insertion based on a Poisson distribution.

The MP1590A can insert random errors using one of the Bitall error insertion timings. The error rate can be set within the range of  $0.1 \times 10^{-10}$  to  $1.0 \times 10^{-2}$ . Random errors can be inserted into all areas of the OTU1 and OTU2 frames.

(When fixed rate  $1.0 \times 10^{-3}$  or  $1.0 \times 10^{-2}$  is set for the error rate, random error insertion is automatically selected.)

## 3: Example of a Measurement System Using the MP1590A

### 3-1 FEC Tolerance Test

- FEC Decoder testing

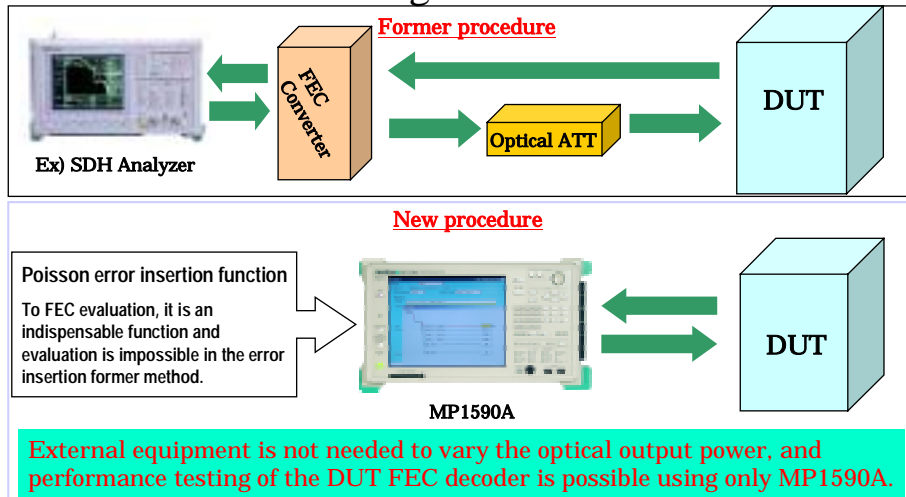


Figure 7 FEC tolerance test

In an FEC tolerance test using a former SDH/SONET measuring instrument such as the MP1570A, an FEC converter and an optical attenuator must be connected between the DUT and the measuring instrument. In this case, the measurement system contains many devices and requires a large installation space. In addition, the test efficiency is reduced because several devices must be controlled simultaneously.

The MP1590A, however, has built-in FEC, random error insertion, and optical attenuation functions. Accordingly, a measurement system can be constructed by using the MP1590A and DUT only. In addition, since the random error insertion function of the MP1590A creates a condition close to that of physical circuits, more accurate FEC tolerance tests can be performed.

### 3-2 Inserting Random Errors into OTU Frames that Support FEC Other Than RS (255, 239)

By using the MP1590A in through mode, random errors can be inserted into an OTU frame that is coded using FEC codes other than RS (255, 239) (the customer's own FEC codes that use the parity check area of the OTU frame).

As shown in Figure 8, the MP1590A and DUT are connected and the MP1590A is set to transparent through mode and FEC OFF. This enables the received OTU frame coded using the customer's own FEC codes to be output from MP1590A Transmitter.

The random error insertion function of the MP1590A then adds errors to the OTU frame. The DUT receives signals with random errors attached and performs error correction using the FEC codes. The FEC performance can thus be evaluated.

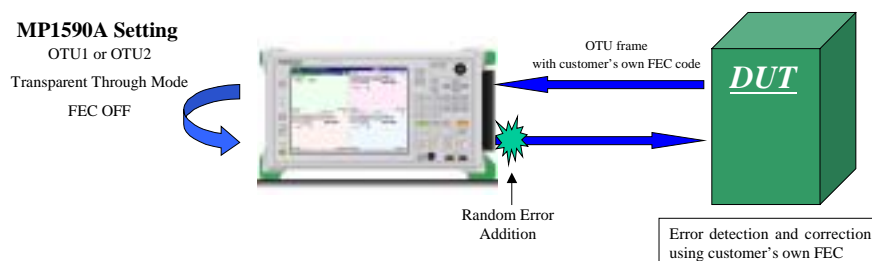


Figure 8 Evaluating FEC using through mode



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Specifications are subject to change without notice.

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