Total Harmonic Distortion Measurements

Total harmonic distortion (THD), generally defined as the ratio of the RMS sum of the amplitudes of harmonic signals of a desired fundamental to the RMS sum with the fundamental included, is a popular measure of nonlinearity of a device. Often, it is desired to evaluate this quantity over a sweep of (fundamental) frequency or power, and as such it can sometimes be complicated to set up since many measurements are needed. In the MS464xB series VectorStar VNAs, multiple source control (Option 7), coupled with the standard equation editor, can give one considerable flexibility in defining this measurement while providing a relatively simple way to orchestrate all of the needed acquisitions. This paper will look at some of the variations on defining THD and some different ways that the MS464xB can be configured to make them. We will begin with some definitions, cover two-port DUT measurements, and then explore variations with differential DUTs.

THD Definitions and Calculation Approach

THD has its origins in audio engineering and later has been broadly applied to RF problems so there are a number of different definitions in use.

$$THD_{fund\_norm} = \frac{\sqrt{|b_{H=2}|^2 + |b_{H=3}|^2 + |b_{H=4}|^2 + \cdots}}{|b_{fund}|} = \frac{\sqrt{\sum_{k=2}^{N} |b_{H=k}|^2}}{|b_{fund}|}$$

Where b denotes a test channel signal and has units of root-power (or equivalently in this particular case, voltage, since reference impedances will cancel in this equation). The ‘H’ denotes the harmonic number of the term (H=2 means 2nd harmonic, etc.) Thus, THD in this case is the RMS sum of harmonic voltages (considering only integer harmonics) divided by the fundamental voltage. Thus, if we want THD for fundamental frequencies of 1 GHz and 2 GHz and we only care about the 2nd through 4th harmonics (higher order ones are in the noise floor), then eight total measurements are needed.

Another definition is to normalize against the total RMS sum instead of just the fundamental. In this case, the equation would be:

$$THD_{RMS\_norm} = \frac{\sqrt{\sum_{k=2}^{N} |b_{H=k}|^2}}{\sqrt{\sum_{k=1}^{N} |b_{H=k}|^2}}$$

The denominator includes the fundamental (H=1). This usage is less standard but has the property that the value can never exceed unity (or 100%, as it is often expressed).

Many other permutations are possible for both of these basic forms

- The numerator includes noise power RMS summed in some bandwidth. This is not common in RF applications.
- The numerator includes summing fractional harmonics. This can be useful for some particular classes of nonlinearities or when one wants to take into account drive impurities. The drive impurities are usually not included in the normalizing denominator but there are even exceptions to this.

For the examples in this paper, we will use the fund_norm definition but the extensions to the other non-noise-based definitions are fairly straightforward.
Since these equations all require absolute unratioed parameter measurements, receiver calibrations are important to accuracy. Details of this calibration class are discussed in the MS4640B Calibration and Measurement Guide but the basic concept is to transfer absolute power accuracy from the source to the receiver of interest with a thru connection. While one can rely on the factory ALC calibration for that source power accuracy, a user power calibration (using a power meter) at the exact frequencies of interest will reduce uncertainty (from potentially several dB to a fraction of a dB, typically). The power calibration and receiver calibration should be performed over a range including all harmonic frequencies that may be relevant for the DUT. In choosing which harmonics to include, one may want to consider the impact of different relative harmonic levels:

- If a particular DUT harmonic is at -40 dBc relative to the fundamental, it will contribute less than 1 percentage point to THD
- If a particular DUT harmonic is at -60 dBc relative to the fundamental, it will contribute less than 0.1 percentage point to THD

The sequence of power calibrations and receiver calibrations is sketched in Fig. 1 for the case where the DUT output will be connected to port 2 of the VNA and port 1 is being used as the source.

The power calibrations and receiver calibrations automatically interpolate and extrapolate in frequency, but uncertainties can increase since the receiver frequency response has to be modeled and fit in the interpolation process. The power calibrations will extrapolate when different power levels are used after calibration, but this can increase uncertainties sometimes. One can use the same power calibration for DUT drive and for the receiver calibration, but if the DUT drive needs to be very low (< -30 dBm), the IF bandwidth should be reduced for the receiver calibration so signal-to-noise related uncertainties do not increase.

Note that this process can be performed on a standard 2-port VNA, on a 4-port VNA, or on one of the broadband/mm-wave versions (ME7838x). The same power calibration and receiver calibration processes apply, although for the broadband and mm-wave systems, different and/or additional power meters may be required to cover the frequency range.

Now that the VNA receiver has been calibrated to accurately measure the quantities in the equations above, the next step is to set up the instrument to measure the harmonics of interest (and the fundamental) and to compute THD. Depending on the amount of flexibility and intermediate data desired, there are two popular setup approaches: the multichannel and the single channel methods.
**Multichannel Measurement Concepts**

The multichannel method provides the maximum amount of intermediate data and the most flexibility in changing measurement conditions for each harmonic but requires a bit more configuration work and does occupy several channels of the VNA.

The central idea in this method is that each channel of the VNA is used to measure one harmonic. Each channel will be sweeping the same number of points (to make the THD calculation simpler) but over the needed range of frequencies for that particular harmonic. The same receiver calibration can be used in all channels (if it was performed over the full frequency range as discussed earlier). The harmonic data can be saved for each harmonic independently if desired and the acquisition parameters can be tailored for each harmonic (e.g., different IF bandwidths, etc.). In sensitivity experiments, the DUT drive level for each harmonic can be varied separately (although this will distort the THD definition, it can be useful for quickly checking sensitivities of each harmonic independently).

To show the setup process, we will consider a simple example of a 1 GHz to 20 GHz amplifier (fundamental range) where only the 2\textsuperscript{nd} and 3\textsuperscript{rd} harmonics are of interest. We will measure the fundamental in channel 1, the 2\textsuperscript{nd} harmonic in channel 2 and the 3\textsuperscript{rd} harmonic in channel 3. We will compute THD in channel 4 for clarity but this can also be done in one of the other channels.

To begin, a power cal and receiver cal covering 1 GHz to 60 GHz was performed and the receiver calibration saved. Channel 1 was set up using multiple source control to cover 1 GHz to 20 GHz looking at the fundamental (default equations, the source and receiver both point to the frequency list). Since this is the fundamental, one technically does not have to use multiple source control (since standard mode would, of course, always look at the fundamental) but we did it this way for clarity.

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*Figure 2. The Channel 1 Multiple Source Equations are Shown Here. The Equation Values are All at Default so Both the Source and Receiver are Looking at the Same Frequencies and in the Range 1 GHz to 20 GHz.*
This setup was saved (.chx) and copied into channels 2 and 3. Then we only need to edit the harmonic number in those channels. The resulting equations are shown in Fig. 3. Note that the receiver is now pointing to the 2\textsuperscript{nd} or 3\textsuperscript{rd} harmonic. The Receiver Source equation is used to index the receiver cal to those same harmonic frequencies.
Although it does not need to be in a separate channel, the equation for THD was placed in channel 4 for this example (using the fundamental normalization definition) and expressed in percentage (%) terms. The equation editor screen is shown below. Note the POW function (raise to a power and the argument is 2) is used for squaring and the SQRT function is used to complete the RMS harmonic sum in the numerator. ‘Corrected’ format is used so the receiver calibration is applied correctly and that is denoted by the small c in the arguments (‘Ch2.cTr1’ means use the corrected format of trace 1 data in channel 2). The corrected parameters are complex so the ABS function is needed to take the magnitude. The 100* prefix converts the linear result to percentage terms.

![Equation Editor Screen](image)

*Figure 4. The Equation Used for THD Computations in this Multichannel Example.*
The result is shown in Fig. 5 and one can see a peak THD of slightly over 4%. This value intuitively makes some sense since the second harmonic is >-30 dBc midband so THD would have to exceed 3% just based on the 2nd harmonic. The x-axis in channel 4 is not used since the equation editor is being applied to the only trace. Note that an alternate trace label was used (THD(%) for clarity. A response parameter has to be assigned to the trace (b2/1|P1 again in this case) but it is not used in the analysis when the equation editor is turned on.

For the computation, a real graph type was used. The output of the equation was linear and real so a linear magnitude graph type could also have been selected. An imaginary graph type would read 0 and a log mag graph type would display 20*log10 of the THD value. The latter might be useful if the 100* was not in the equation (10% THD is 0.1 linear so a log mag representation would be -20 dB).

We did not do it in this example, but the IF bandwidth (or source power) could have been independently changed in channels 1 to 3 to alter the acquisition behavior for each element. The settings in channel 4 would not matter since that data is not being used but the point count must be same so the equation plotting will work (error messages will be generated if not). The data can be saved from channels 1 to 3 to get the fundamental or harmonic powers in absolute terms (because of the receiver calibration, the graphs in those channels can be read in dBm terms).
Single Channel Measurement Concepts (streamlined)

The single channel method does not provide as much intermediate data and it is a little more complicated to adjust acquisition parameters on a per-harmonic basis but the configuration is quick to set up and only one VNA channel is used.

The premise here is to construct a sweep covering all of the harmonic frequency ranges of interest in a single channel and then use some special features of the equation editor to separate out the harmonics during the computation process. In its simplest state, all harmonics will be acquired with the same IF bandwidth and drive power. Variations like those discussed for the multiple channel method can be implemented using segmented sweep where control over the measurement variables is possible over portions of the sweep.

Again, we will consider an example where only the 2\textsuperscript{nd} and 3\textsuperscript{rd} harmonics are of interest and the fundamental range is 1 GHz to 20 GHz. A power calibration and receiver calibration covering 1 GHz to 60 GHz was again the starting point but a single channel was used. Three bands were created in multiple source control to look at the fundamental, 2\textsuperscript{nd} harmonic and 3\textsuperscript{rd} harmonic. A single frequency list was used with 573 points (100 MHz step size) going from 1 GHz to 58.2 GHz. These ‘display’ frequencies are converted into the desired source and receiver frequencies using the multiple source equations as shown in Fig. 6. The first band trivially looks at the fundamental and the default equations are used, and this includes 191 points.
The second band will also include 191 points (starting at 20.1 GHz display frequency and stepping by 100 MHz) but we want the source to again sweep 1 GHz to 20 GHz. Thus the source 1 equation becomes $F - 19.1$ GHz (where $F$ is the display frequency). We want the receiver to look at 2x the source frequency so the same equation with a multiplier of 2 is employed. The last display frequency of this band is 39.1 GHz (source frequency $= 39.1 - 19.1 = 20$ GHz).

The third band will also include 191 points (starting at 39.2 GHz display frequency and stepping by 100 MHz) but we want the source to again sweep 1 GHz to 20 GHz. Thus the source 1 equation becomes $F - 38.2$ GHz (where $F$ is the display frequency). We want the receiver to look at 3x the source frequency so the same equation with a multiplier of 3 is employed. The last display frequency of this band is 58.2 GHz (source frequency $= 58.2 - 38.2 = 20$ GHz).

There are many different ways that this sweep could have been set up. The critical consideration is that there must be the same number of points used in each band so that the equation can be processed on a point-by-point basis.

The equation entry is shown in Fig. 7 and the structure is similar to that shown in Fig. 4. The ABS function is again used to compute magnitude and the POW and SQRT functions are used to compute the RMS sum. Now we must also use the SUBSET function to pull out the relevant portions of the sweep data. The SUBSET syntax is SUBSET $(a,b,trace)$ where $a$ is the index of the first point of the range (indices start at 0), $b$ is the index of the last point of the range, and $trace$ is the relevant data vector.

In our example, the fundamental data is contained in points 0 through 190, the 2$^{nd}$ harmonic data is contained in points 191 to 381, and the 3$^{rd}$ harmonic data is contained in points 382 to 572. The equation shown below then processes all of these variables that are now vectors of length 191. As a result, all of the output data is contained in the first 191 points and the remaining plotted values will be zero.

![Figure 7. The Equation Editor Xcreen for this Single Channel Example is Shown Here. The Extended Display was Enabled so the Entire Equation Would be Visible.](image)
The THD data is shown in trace 2 of Fig. 8 (DUT in the same family as that used for Fig. 5). The incoming data (fundamental, 2nd harmonic and 3rd harmonic) are shown in trace 1 and the transitions between multiples are the obvious steps. All of the points are indexed between the sub-bands to enable the calculation.

![Figure 8](image)

**Figure 8.** The THD Measurement Result is Shown Here in Trace 2 in the First 191 Points. The Combined Fundamental-2nd Harmonic-3rd Harmonic Sweep is Shown in Trace 1.

It was not used in this example, but modifications to the acquisition parameters can be done in this method using segmented sweep. As an example of this, suppose the harmonic levels were relatively low and it was desired to use lower IF bandwidths during those measurements to improve the signal-to-noise ratio (but not waste the time in doing the same thing for the fundamental). A segmented sweep table like that shown in Fig. 9 could be employed where 1 kHz IF bandwidth is used for the fundamental, 500 Hz is used for the 2nd harmonic and 200 Hz is used for the 3rd harmonic. This kind of setup would be most useful when a large number of measurement points are needed for a particular application.

![Figure 9](image)

**Figure 9.** A Possible Segmented Sweep Table is Shown for the Conditions of the Single Channel Method Example that Uses Different IF Bandwidths for the Different Harmonics. Other Parameters can also be Isolated by Band.
**Differential Measurements: Single-Ended Input, Differential Output Devices**

The same measurement concepts, and setups for the most part, apply to the differential output case. The following discussion will assume the use of a 4-port VectorStar but it is possible with a 2-port unit, with one of the loop options (51, 61 or 62).

The only real difference from the earlier sections of this document is that two receiver responses are now needed as are receiver calibrations on two ports. A single power calibration can be used to transfer to the two receiver calibrations. The fundamental norm equation becomes:

\[
THD_{d,fund,norm} = \sqrt{\frac{b_{d,H=2}^2}{b_{d,fund}} + \frac{b_{d,H=3}^2}{b_{d,fund}} + \frac{b_{d,H=4}^2}{b_{d,fund}} + \cdots} = \sqrt{\frac{\sum_{k=2}^{N} b_{d,H=k}^2}{b_{d,fund}}}
\]

Where statistically:

\[
|b_{d,x}| = (|b_{2,x}| + |b_{4,x}|)/2
\]

And we assume the two single-ended ports being used are ports 2 and 4 and ‘x’ can be any of the harmonics of interest or the fundamental. This simple treatment assumes the RF common mode output is small (easy to check) and avoids having to add phase-based receiver calibrations.
The sweep routines discussed in the earlier sections are unchanged but now additional traces must be added to capture the data for the second receiver. Receiver calibrations were performed for b2 and b4 using a through connection to port 1 (sequentially). Using the multi-channel approach, an equation definition and an example measurement are shown in Figs. 10 and 11.

The fundamental responses on the two output ports were relatively symmetric but the harmonic responses were less so. This could be due to the intrinsic circuit design or may be a result of the packaging used. One might argue whether the ‘simple’ approach of magnitude addition is appropriate in this case as there are hints of common mode energy (particularly at the harmonics) but this metric may be more useful in practice since it establishes an upper bound that might be reached with even a more asymmetric implementation layout where the circuit might be used.

![Equation editor screenshot](image.png)

*Figure 10. The Extended-View of the Equation Used for the THD Computation in this Section. The Two Outputs are Assumed to Add in a Voltage Sense in this Case which Generally Leads to a Conservative THD Value. The Extended View of the Equation is Shown Here Since the Entire Equation is Not Visible on the Main Equation Editor Screen.*

Note that the $\sqrt{2}$ in the $|bd,x|$ equation in this document was skipped in the equation editor since it was common to both numerator and denominator and would cancel.
Figure 11. The Multi-Channel THD Measurement is Shown Here for a Differential-Out Amplifier. As in the Single-Ended Example of Fig. 5, ch1 is the Fundamental Response, ch2 is the 2nd Harmonic and ch3 is the 3rd Harmonic. The THD is Computed in ch4 and Expressed in % Terms (peak value of about 2% at the power level used here).

If the common-mode output is high (e.g., > -10 dBC), then an explicit vectorial capture may be needed and the more explicit response term is:

\[ |b_{d,x}| = \left| b_{2,x} - b_{4,x} \right| / 2 \]

The phase correction is most simply accomplished using trace memory. Since the receiver phase responses can only easily be captured in a ratioed form, one approach is to connect a thru line (as was done for the receiver calibrations earlier) between ports 1 and 2 and save \( b2/a1 |P1 \) to trace memory (for example) trace 3. Then, make the thru connection between ports 1 and 4 and save \( b4/a1 |P1 \) to memory (for example) trace 4. Traces 3 and 4 should be configured as real and imaginary graph types so the equation can make use of formatted variables. This must be done for all three measuring channels separately and, for the harmonic channels, the source must be set equal to the receiver multiplier so it is non-converting (only for this step, then change the source equation back to the default when done). It is also convenient to change the Tr1 and Tr2 definitions to \( b2/1 |P1 \) and \( b4/b2 |P1 \), respectively.
\[ |b_{d,x}|^2 \propto |b_{2,x} - CAL \cdot b_{4,x}|^2 \]
\[ = |1 - CAL \cdot b_{4,x}/b_{2,x}|^2 \cdot |b_{2,x}|^2 \]

Where \( CAL \) is a phase function of the ratio of the two memory traces just stored:

\[
CAL = e^{i \angle (Mem_{b2}/Mem_{b4})}
\]

Thus, the equation takes the ratio of \( b_4/b_2 \) and corrects it by the receiver path length differences for ports 2 and 4 (represented by \( CAL \)). The modified THD equation then becomes that shown in Fig. 12. Again, the \( \sqrt{2} \) terms were skipped in equation entry since they are common to both numerator and denominator.

Figure 12. The Multi-Channel THD Measurement is Shown Here for a Differential-Out Amplifier. As in the Single-Ended Example of Fig. 5, ch1 is the Fundamental Response, ch2 is the 2nd Harmonic and ch3 is the 3rd Harmonic. The THD is Computed in ch4 and Expressed in % Terms (peak value of about 2% at the power level used here).
The result of this process when applied to the DUT of Fig. 11 is shown in Fig. 13. One may note the THD is slightly decreased from that in Fig. 11 which is not unusual since the simplified process provides a conservative estimate for total harmonic energy.

Figure 13. The THD for the Example DUT When Handling the Phasing of the Output Signals is Shown Here in Channel 4. The Peak Value is Slightly Reduced from That in Fig. 11 Due to More Accurate Handling of the Differential Harmonic Energy.
Differential Measurements: Fully Differential Devices

Moving now to fully differential devices, the receiver configuration and calculation process is the same as that discussed in the previous section but now the stimulus side must be addressed. The use of an external balun is a possibility and is the only choice if a single source (non-option 31) VectorStar is being used. The balun may also be a recommended choice, even with a dual source (Option 31) VNA, if the DUT is far from the instrument or requires complex fixturing. With Option 31, however, phase coherence and phase control can be used to create true differential drive. There are many ways to accomplish this in VectorStar (see the Calibration and Measurement Guide for more details), but the simplest for THD measurements may be to just engage double-active drive from within multiple source control and rely on the factory phase calibration to get the desired phase relationship.

To set up this state, see the internal source control submenu (within multiple source) illustrated in Fig. 14. Both sources should be set to ‘Active’ and a selection must be made of which ports will drive as a pair and 1 to 3 is a common choice as shown in the figure (leaving ports 2 and 4 to receive the DUT outputs as used in the previous section). Phase Synchronization should also be set to ‘ON’. When these selections are made, a phase offset field will be available on the Frequency menu where the relationship between the sources can be set. This value is calibrated to the instrument ports so cables that are attached should be of the same length.

![Internal Source Configuration Submenu](image)

*Figure 14. The Internal Source Configuration Submenu within Multiple Source Control is Shown Here. This Sets Up the Dual Source Drive that Can be Used in Absence of an Input Balun for the DUT.*

On Channel 1, the source-ref-plane mode of true mode stimulus can also be employed. The factory phase calibration is referenced to the VNA test ports so, again, some care in cable length equalization may be needed.
The equations and processing related to the receiver side is unchanged. A measurement example on a different DUT is shown in Fig. 15. There is some additional distortion cancellation in this DUT which may be related to it being driven symmetrically.

**Summary**

Total harmonic distortion is a nonlinearity metric that is commonly used and can be orchestrated in a number of different ways on the MS464xB VectorStar VNA to optimize speed of setup, flexibility, or access to intermediate measurements. All of these are relatively straightforward to configure using multiple source control and the Equation Editor.
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