FEC (Forward Error Correction) Performance Evaluation Method and Poisson Error Generator

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1. Introduction

Generally, correction of random errors (errors occurring in circuits) uses error-correcting code classified as block code. The ITU-T G.709 Optical Transport Network (OTN) Forward Error Correction (FEC) code uses the Reed Solomon codes (RS255 and RS239). Since the Reed Solomon codes are block codes, generation of pseudo-random errors makes it impossible to evaluate FEC decoder performance by comparing the error correction performance with the theoretical curve for example. The theoretical curve shown in Fig. 1 plots the random error occurrence and shows that FEC performance cannot be evaluated correctly even when errors are inserted at some bit rate and the same interval. Accordingly, it is necessary to generate errors randomly and to create error correction and non-correction conditions so the long-term random error rate is satisfied while the short-term error rate varies in line with the set value. This approximates the actual conditions of an in-service network and is a suitable condition for evaluating FEC performance.

This type of random error generation device is called a Poisson error generator.

However, the random errors generated by a Poisson error generator are thought to be dependent on the evaluation method and do not actually fit the Poisson distribution. Accordingly, we need an objective method for evaluating whether or not random errors generated by a random error generator fit the Poisson distribution. There are several well-established methods for testing goodness of fit to the Poisson distribution but this paper proposes using the χ^2 test as the method recommended by ITU-T O.182 (OTN Measurement Standards).

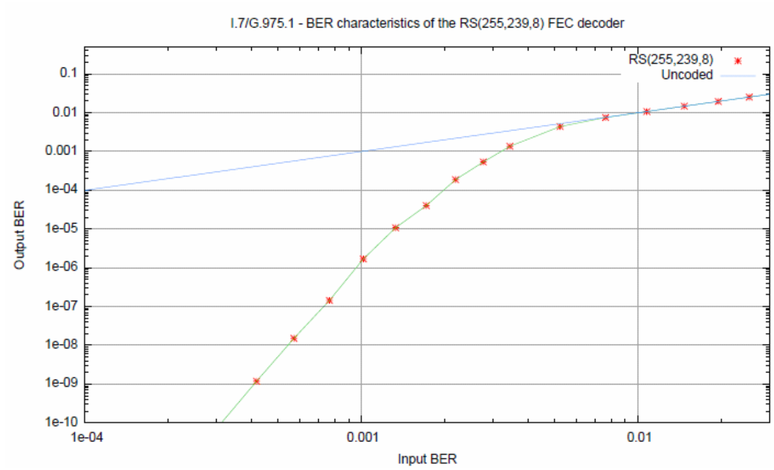


Figure 1 Output-Input BER

2. Principle of Random Error Insertion

The previously described random error insertion method creates error occurrence conditions that approximate an in-service circuit. In comparison to methods that insert errors as constant interval or intentional sequence, inserting random errors will result in some time slots with no errors, but the long-term error rate will still be satisfied (Fig. 2).

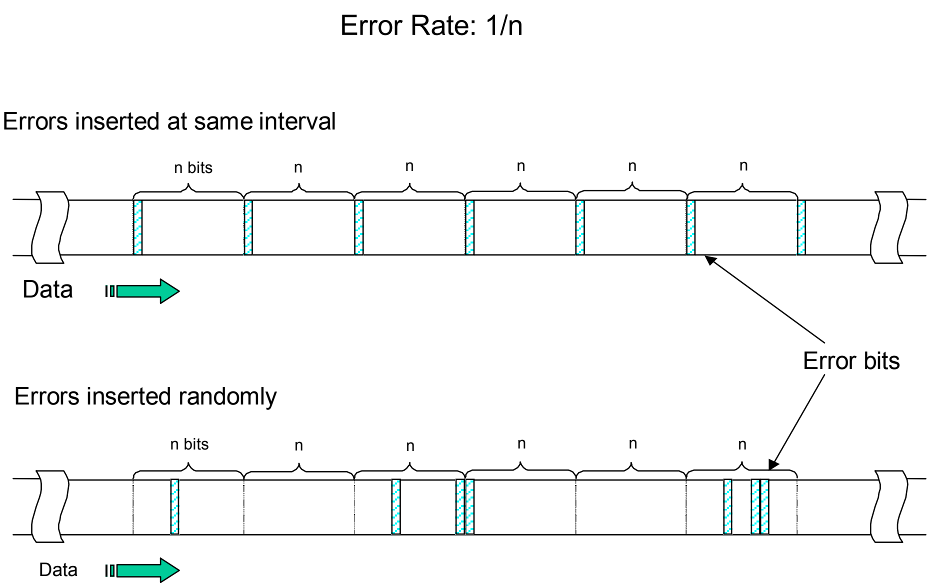


Figure 2 Error Rate Insertion Methods

2.1 Binomial and Poisson Distributions

In a test with two possible outcomes A and B, the result of each repeated trial is independent of the result of the previous trial and is a constant that depends on the probability (p). This type of trial testing is known as a Bernoulli Trial. As a simple example, when an Othello counter (black on one side and white on the other) is repeatedly thrown in a Bernoulli Trial on a surface board, assuming the counter is equally weighted without bias, the probability of either the white or black side being uppermost is 0.5 (1/2).

When a trial with possible outcome A and B is repeated n times, the probability of obtaining the A result k times is given by the following equation (1), where μ is the mean value and σ^2 is the distribution. This type of probability distribution is called a binomial distribution.

$$f(n,k) = \binom{n}{k} p^k (1-p)^{n-k} \qquad \begin{cases} \mu = np \\ \sigma^2 = np(1-p) \end{cases} \qquad (1)$$

Explaining Equation (1) as an example of error generation, the distribution of k bit errors in n bits follows the same probability distribution (k/n). However, direct computation by inserting values into Equation (1) is extremely difficult due to the increasing value of n. However, substituting λ for np and making λ constant shows that p and n are inversely related with p becoming small as n increases and vice versa. Considering the most extreme distribution of the binomial distribution (n = ∞, p = 0) yields the following equation (2).

$$f(k;\lambda) = e^{-\lambda} \frac{\lambda^k}{k!} \qquad \begin{cases} \mu = \lambda \\ \sigma^2 = \lambda \end{cases} \qquad (2)$$

The probability distribution given by Equation (2) is called the Poisson distribution. Since λ is a fixed value, the distribution is expressed by the variable k. Due to the relationship between the Poisson and binomial distributions, substituting np for λ yields

$$f(k;np) = e^{-(np)} \frac{(np)^k}{k!} \qquad (3)$$

Poisson distribution of probability function k for fixed values of np

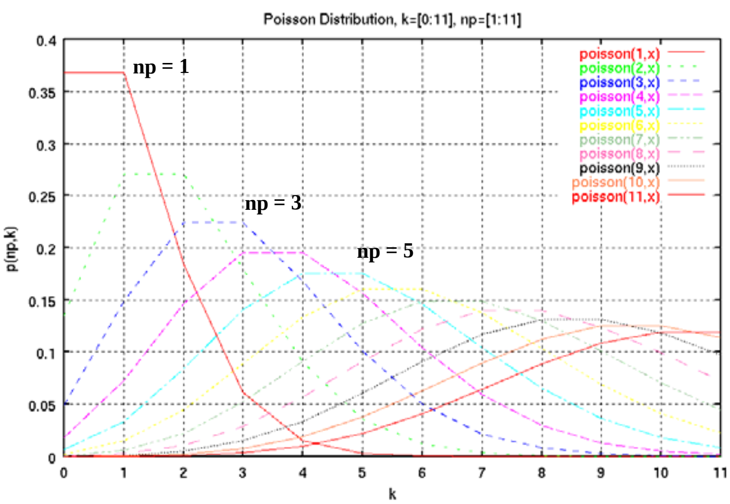


Figure 3 Poisson Distribution

Figure 3 shows the Poisson distribution as a graph. This graph shows the distribution of probabilities for generating a phenomenon k times versus the average value of np. For example, for a baseball player with a batting average of 3, the chance of hitting the ball in 10 attempts is $\lambda = np = 10 \times 0.3 = 3$ hits. However, sometimes the baseball player will hit the ball 8 out of 10 times and at other times only 1 out of 10 times. In these circumstances, the probability of hitting the ball 5 out of 10 times is obtained from equation (3) as:

$$f(k = 5; np = 3) = e^{-(3)} \frac{(3)^5}{5!} = 0.1008188......... \qquad (4)$$

In other words, there is about a 10% chance of hitting the ball 5 out of 10 times.
This result is the probability distribution expressed by “poisson(3,x)” in Figure 3. Using the “poisson(3,x)” plot, since this player hits with an average of 3, the peak of the plot is at k = 3 and the probability of getting 3 out of 10 hits is 22.4%. In addition, we can see that the probability drops whether the hit count rises or falls.

2.2 Poisson Distribution and Error Rate

Considering the relationship between random error and the Poisson distribution, p represents the element error rate (error rate). np represents the mean error but count μ for a frame length of n bits. For example, when p is fixed, $n = \mu / p$.
Figure 4 shows equivalent case for the error rate example in Figure 3. When the error rate p is fixed at p : $1\text{x}10^{-3}$ and the average error bit count is increased as 1, 2, 3, ... bits, the frame length n becomes $1\text{x}10^{3}$, $2\text{x}10^{3}$, $3\text{x}10^{3}$, ... bits.

Poisson distribution of probability function k for fixed values of np

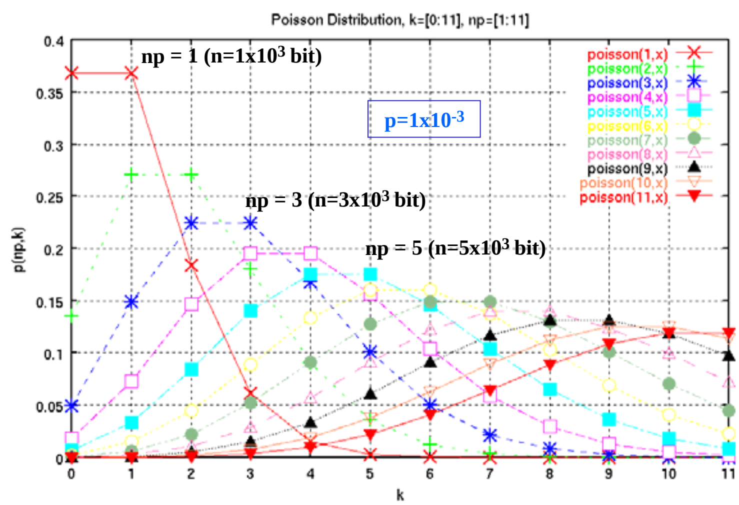
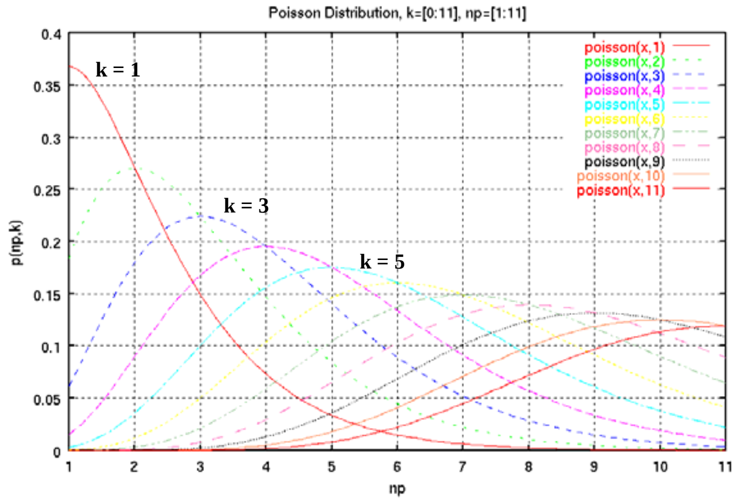


Figure 4 Distribution when p = $1\text{x}10^{-3}$

k represents the error bit count. When the average error bit count np = 1 bit (poisson(1,x)), k = 1 the peak error probability is at k = 1 bits and there is a lower probability of 2 bit errors (k = 2) in $1\text{x}10^{3}$ bits. Similarly, at np = 5 bits (poisson(5,x)), the peak error probability is at k = 5 bits and the probability of errors of less than or more than 5 bits decreases.
Random errors can be generated based on this probability distribution.
Looking at the relationship between error rate and Poisson distribution from another angle, Figure 5 shows the Poisson distribution when error bit count k is fixed and average error bit count np is varied.

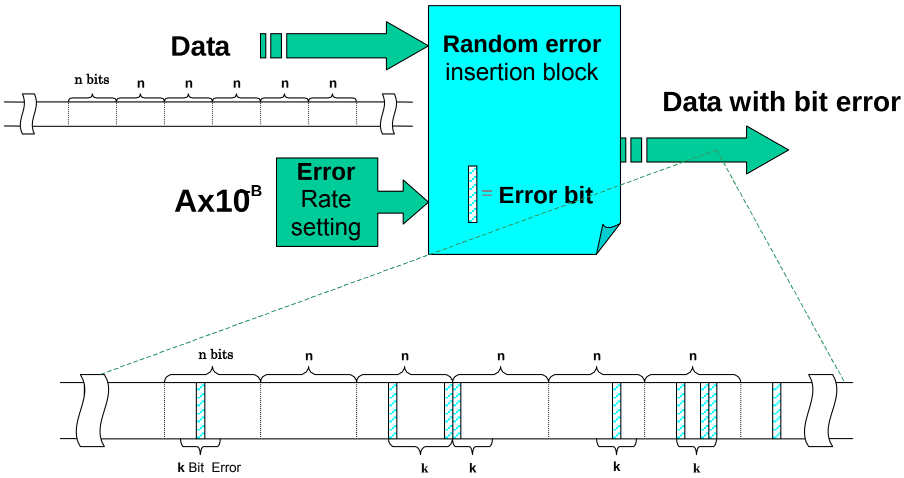
Poisson distribution of probability function np for fixed values of k



$p = 1x10^{-3}$:	n = 1000	2000	3000	4000	5000	6000	7000	8000	9000	10000	11000
$p = 1x10^{-5}$:	$n = 1x10^5$	$2x10^5$	$3x10^5$	$4x10^5$	$5x10^5$	$6x\ 10^5$	$7x10^5$	$8x10^5$	$9x10^5$	$10x\ 10^5$	$11x10^5$
$p = 1x10^{-7}$:	$n = 1x10^7$	$2x10^7$	$3x10^7$	$4x10^7$	$5x10^7$	$6x\ 10^7$	$7x10^7$	$8x10^7$	$9x10^7$	$10x\ 10^7$	$11x10^7$

Figure 5 Poisson Distribution with Variation of np

poisson(x,1) in Figure 5 shows the k = 1 bit error probability versus np. When the error rate is fixed at $p = 1x10^{-3}$ (average of 1 bit error in 1000 bits), the peak is at n = 1000 bits. The 1 bit error rate in 2000 bits at np = 2 is lower than at np = 1. Similarly, poisson(x,3) indicates the k = 3 bit error probability distribution; when the error rate is fixed at $p = 1x10^{-3}$, the peak is at n = 3000. Figure 6 shows an image of actual random error insertion.



The distribution of k error bits inserted into each block
of n bits follows a Poisson distribution.
Figure 6 Image of random error insertion

Figure 6 Image of Random Error Insertion

When data is split into block of n bits, the distribution of the bit error count k inserted into each block follows a Poisson distribution. Using the value np of the average error bit count, we can see how error bit count k is distributed using the graphs in Figure 4.

3. Applicability of χ^2 Test to Poisson Distribution Goodness of Fit

Sometimes, the random error generator called a Poisson Error Generator does not generate errors fitting the Poisson distribution. Consequently, we require an objective method for evaluating the distribution of random errors generated by the random error generator. The methods for evaluating probability distributions are called tests of goodness of fit and objectively evaluate whether the obtained probability distribution matches the hypothesis. There are several goodness of fit test methods but this White Paper explains the χ^2 Test of Goodness of Fit used by ITU-T O.182 (OTN Measurement Standards).

The χ^2 test is described as a hypothesis test because it tests whether or not the sample of finite observed data accurately describes the hypothesis. The χ^2 test is one of many hypothesis methods for testing goodness of fit but it is a typical method. Like other test methods, the χ^2 test is not a universal test for goodness of fit and it be used for all distributions. Fortunately, however, it is one of the best methods for testing the goodness of fit for Poisson distribution and is frequently used when the assumed distribution is a Poisson distribution.

As already mentioned, λ, the only parameter of the Poisson distribution can be chosen freely by determining any value for the observation time n. Actually, when testing goodness of fit to the Poisson distribution, the optimum value for λ is given by the following relationship.

$$5 \le \lambda \le 20 \qquad (5)$$

It is especially important to satisfy the lower limit of the formula.

On the other hand, the upper limit does not always need to be satisfied because it is obtained from the observation time limit. If the situation permits, λ does not necessarily need to be restricted by the upper limit of (5) if no problems are caused by increasing the observation time.

When the value of λ is small, the Poisson distribution is skewed to the left and the number of observations is reduced as a result. Here, the observed frequency k is the error occurrence frequency (including 0) for observation time n. The details of the observed frequency are explained later.

When testing goodness of fit, the number of observed items should be at least 5. Although this value is based on empirical evidence, it is necessary to maintain the reliability of the result of the hypothesis test at a fixed level. In fact, since the χ^2 test is based on large sample theory, small sample sizes reduce the test reliability.

On the other hand, if the value of λ is larger than necessary, the number of observations increases as a result while the frequency of each item becomes small. Of course, although the frequency of each item can be increased by increasing the number of observations, the observation time also increases as a result. When testing goodness of fit using the χ^2 test, the frequency of each item should be at least 5. This is another limitation on using the χ^2 test that is required to maintain the reliability of the goodness of fit test result at a fixed level.

4. Principle of χ^2 Test

Letting $f_1, f_2, \ldots, f_j$ and $e_1, e_2, \ldots, e_j$ be the observed frequency and expected frequency for *N* experiments where j is the number of possible outcomes, the value given by the equation

$$\chi^2 = \sum_{i=1}^{j} \frac{(f_i - e_i)^2}{e_i} \tag{6}$$

approaches the χ^2 distribution for $\nu = j - 1 - t$ degrees of freedom as *N* increases where *t* is the number of estimated parameters. Generally, when testing goodness of fit for the Poisson distribution, the parameter λ is estimated from the sample, so degrees of freedom $\nu = j - 2$.
From equation (6), it is clear that χ^2 becomes smaller as the observed sample fits the proposed hypothesis. Normally, this never happens but if the value of χ^2 のfound by the above equation became 0, it would indicate a perfect match between the distribution of the observed sample and the hypothetical distribution. The test of goodness of fit using the χ^2 test is a method for testing the hypothesis based on the value of χ^2 found by the above equation.
Furthermore, since the χ^2 distribution is equivalent to when $\alpha = \nu/2$, and $\beta = 2$ in the gamma distribution, therefore,

$$\chi^2(x) = Ga(x) = \begin{cases} \dfrac{x^{\nu/2-1} e^{-x/2}}{2^{\nu/2}\Gamma(\nu/2)}, & x \geq 0 \\ 0, & x < 0 \end{cases} \tag{7}$$

where ν is the degrees of freedom of the χ^2 distribution. Moreover, $\Gamma(\alpha)$ is the gamma function given by the following equation.

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx \tag{8}$$

Since $\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$, when α is an integer value, $\Gamma(\alpha + 1) = \alpha!$ And the value of the gamma functions can be factoring, explaining why the gamma function is sometimes called a factorial function.
Moreover, since the χ^2 distribution function is also a probability density function, the function area approaches 1, meaning

$$\int_0^\infty \chi^2(x) dx = 1 \tag{9}$$

Calculating the area of the right tail from some point χ_α of the χ^2 distribution (representing the incomplete integration as α*),* then

$$\alpha = \int_{\chi_\alpha^2}^\infty \chi^2(x) dx \tag{10}$$

Because the entire area of the χ^2 distribution function is 1, the value of α given by the above equation represents the area ratio and α is called the significance level. Generally, significance is expressed as a percentage obtained by multiplying α by 100 and this White Paper follows the same convention.
Since the χ^2 distribution is a monotonous and continuous function, $\chi^2{}_\alpha$ can be found from the significance level 100 x α and is called the significance point, critical point, or percent point.

Because it is not always easy to calculate the required significance point from the significance level, many statistics books have appendices containing tables of the χ^2 distribution function, which are used to find the significance point from the significance level and degrees of freedom. Figure 7 shows an example for the χ^2 distribution with 10 degrees of freedom (ν).

The χ^2 test of goodness of fit evaluates the following based on the significance point found from the significance level

$$H_0 = \begin{cases} accept \quad if \;\; \chi^2 \leq \chi_\alpha^2 \\ reject \quad if \;\; \chi^2 > \chi_\alpha^2 \end{cases} \tag{11}$$

where H_0 is the null hypothesis. In other words, if the value of χ^2 found from equation (6) is less than the significance point $\chi^2{}_\alpha$ found from the significance level 100 x α, the null hypothesis is accepted. Conversely, when the value of χ^2 exceeds the significance point, the null hypothesis is rejected.

4.1 Estimating Poisson Distribution Parameter

When testing the goodness of fit for the Poisson distribution, the parameter λ is not usually known so it must be estimated from the obtained sample.

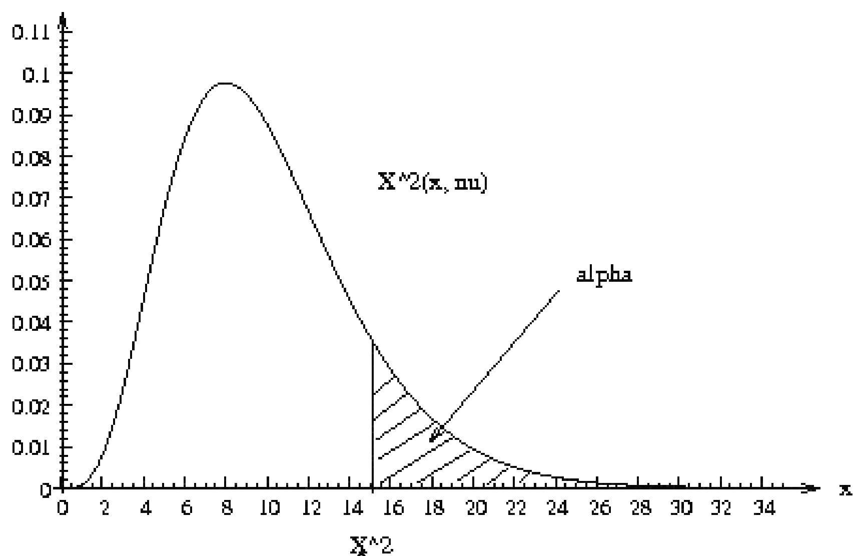


Figure 7 χ^2 Distribution function

In estimating the Poisson distribution parameter, it is known that the maximum likelihood estimation and first moment (or the sample mean) are coincident. Consequently, the Poisson distribution parameter can be estimated simply from the following equation.

$$\hat{\lambda} = \frac{1}{N} \sum_{i=1}^{j} i \cdot f_i \tag{12}$$

where N is the sample size.

When testing the goodness of fit, it does not matter whether the expected frequency is either

calculated from this parameter, or estimated from the observed sample.
The *k*-th probability *pk* for the Poisson distribution calculated based on the determined value of λ is found from the following equation.

$$p_k = e^{-\hat{\lambda}} \frac{\hat{\lambda}^k}{k!} \quad (13)$$

And the *k*-th expected frequency is given by

$$e_k = N \cdot p_k \quad (14)$$

5. Examples of Poisson Distribution Goodness of Fit

This section explains two examples of Poisson distribution goodness of fit. The first example shows a fit at the 5% significance level while the second shows an example that does not a fit at the 5% significance level.

Table 1: Samples

k	Observed frequency f_k	Expected frequency $e_k = np_k$	Probability p_k
5	1	1.184	0.000967
6	9	3.114	0.008704
7	7	7.019	0.006770
8	9	13.842	0.008704
9	20	24.267	0.019342
10	32	38.287	0.030948
11	56	54.916	0.054159
12	78	72.203	0.075435
13	93	87.630	0.089942
14	107	98.756	0.103482
15	112	103.875	0.108317
16	83	102.431	0.080271
17	102	95.065	0.098646
18	75	83.328	0.072534
19	70	69.195	0.067698
20	55	54.587	0.053191
21	40	41.012	0.038685
22	26	29.412	0.025145
23	24	20.176	0.023211
24	16	13.264	0.015474
25	4	8.371	0.003868
26	6	5.080	0.005803
27	5	2.968	0.004836
28	3	1.673	0.002901
29	0	0.910	0.000000
30	1	0.479	0.000967
Total	$\Sigma f_k = 1034$	$\Sigma e_k = 1033.040$	$\Sigma p_k = 1.000000$

Both examples are obtained from an actual Poisson error generator under the same conditions with an element error rate $p_e = 10^{-8}$ and sample size $N > 1000$.
Furthermore, in both examples, the number of average errors occurring in the observation time n was chosen so that $\lambda = 16$.
In other words, the experiment was executed with observation time $n = \lambda / p_e$. Incidentally, observation time means discrete time (number of actual clocks).

5.1 Good Poisson Distribution Fit

The first example (Figure 9) plots a data sample obtained from a Poisson error generator using the data in Table 1.

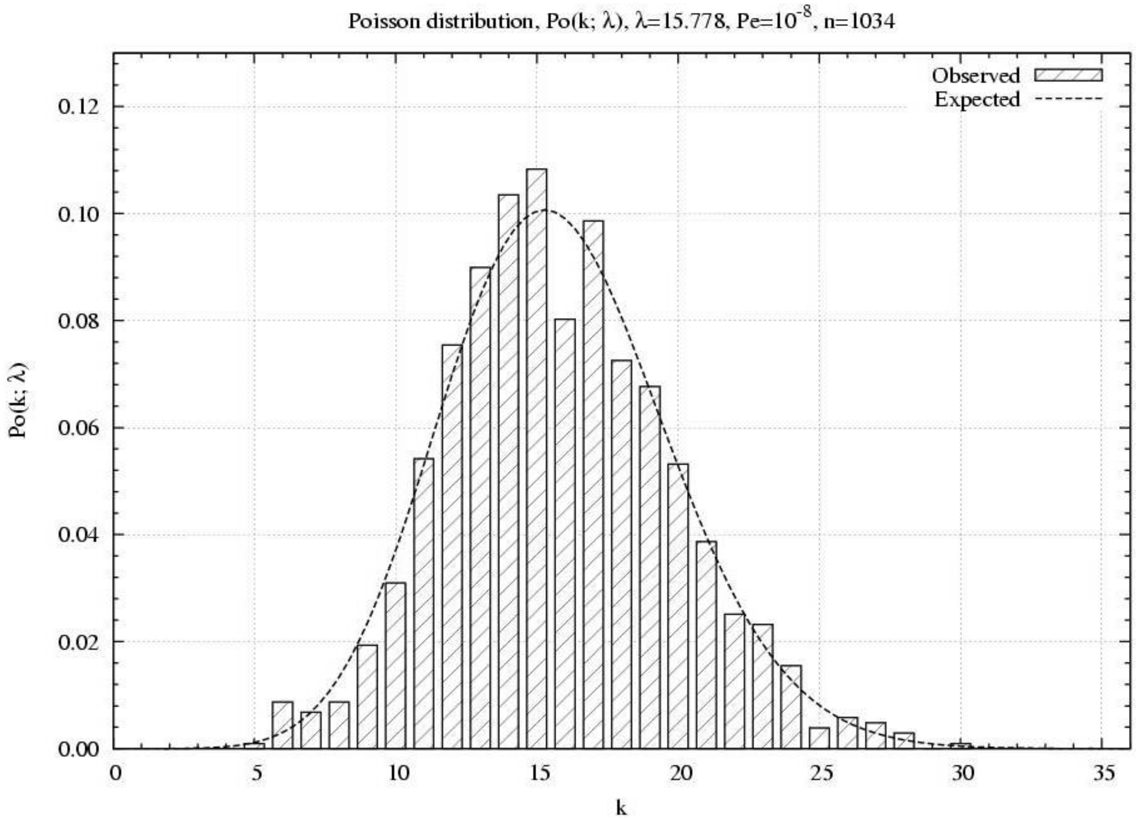


Figure 8 Sample 1

The figure histogram plot shows the probability found from the observed frequencies and the dotted line plots the Poisson distribution function drawn using the sample mean $\hat{\lambda}$. Although the Poisson distribution is actually discrete, it has been plotted as a continuous curve interpolated by the gamma function for easy viewing.

The first column in Table 1 represents the observed number of errors *k* occurring in the observation time n. The second column represents the count, i.e. observed frequency of k errors occurring in the observation time n. The third column number represents the expected frequency for k errors occurring in the observation time *n*, calculated from estimated mean error count $\hat{\lambda}$ for the Poisson distribution calculated using the observed sample. In other words, the expected frequency is calculated by the following equation.

$$e_k = e^{-\hat{\lambda}} \frac{\hat{\lambda}^k}{k!} \cdot N$$

where N is the sample size. The fourth column represents the probability of k errors occurring in the observation time n.
This probability is found easily from the following equation.

$$p_k = f_k / N$$

The bottom row shows the sum for each column. The sum of the second column is sum of the observed frequency, and is also the sample size N. The sum of the third column is the sum of the expected frequency.

Table 2 Sample 1

k	Observed frequency f_k	Expected frequency $e_k = np_k$	Deviation $(f_k - e_k)^2/e_k$
≤ 7	17	11.808	2.283
8	9	13.842	1.694
9	20	24.267	0.750
10	32	38.287	1.032
11	56	54.916	0.021
12	78	72.203	0.465
13	93	87.630	0.329
14	107	98.756	0.688
15	112	103.875	0.636
16	83	102.431	3.686
17	102	95.065	0.506
18	75	8.328	0.832
19	70	69.195	0.009
20	55	54.587	0.003
21	40	41.012	0.025
22	26	29.412	0.396
23	24	20.176	0.725
24	16	13.264	0.565
25	4	8.371	2.282
≥ 26	15	11.578	1.012
Total	$\Sigma f_k = 1034$	$\Sigma e_k = 1034.000$	$\chi 2 = \Sigma (f_k - e_k)^2/e_k = 17.939$

Naturally, there are some small differences between the totals of observed frequency and expected frequency, because the former is sample size and the latter is the sum of the expected frequencies for the k range $5 \le k \le 30$.
The difference is clarified by the following formula.

$$\sum_{k=5}^{30} f_k - \sum_{k=5}^{30} e_k = \sum_{k=0}^{4} e_k + \sum_{k=31}^{\infty} e_k = N \sum_{k=0}^{4} e^{-\hat{\lambda}} \frac{\hat{\lambda}^k}{k!} + N \sum_{k=31}^{\infty} e^{-\hat{\lambda}} \frac{\hat{\lambda}^k}{k!}$$

Here, note that the expected frequencies for $k \leq 6$ and $k \geq 27$ are less than 5.
When testing goodness of fit using the χ^2 test, as mentioned the reliability of the test result deceases when the expected frequency is less than 5. Therefore the table must be rearranged. Table 2 shows the result of combining the row $5 \leq k \leq 6$ in Table 1 with $k = 7$, and the row 27 $k \leq$ 30 with $k = 26$. The fourth row in Table 2 shows the deviation between the observed and expected frequencies. The sum of this deviation is χ^2.
The results of performing this goodness of fit based on this table are shown in Table 3. With this sample, the null hypothesis is accepted at the 5% significance level.

Table 3 Sample 1 Goodness of Fit Test Result

Item	Symbol	Value
Sample size	N	1034
Estimated λ	$\hat{\lambda}$	15.7776
Degrees of freedom	ν	18 (k = 7, ..., 26)
Chi-square	χ^2	17.9395 (tail area = 45.96%)
Significance level	α	5.0%
Significance point	χ^2_{α}	28.8693
Hypothesis	H_0	Accept

5.2 No Poisson Distribution Fit
The second example plots a data sample obtained from a second Poisson error generator. Table 4 shows the sample for this. The values in the table have the same meaning as the previous example. The plot in Figure 9 is based on the data in Table 4 and the meanings of the parts of the plot are the same as the previous example.
Here, in Table 4, note that the expected frequencies for $k \leq 6$ and $k \geq 27$ are less than 5. Consequently, the same combination as in the previous example is required. Table 5 shows the result of combining the row $0 \leq k \leq 6$ with $k = 7$ and the row for $27 \leq k \leq 45$ with $k = 26$. Table 6 The results of performing this goodness of fit based on this table are shown in Table 6. With this sample, the null hypothesis is rejected at the 5% significance level.

Table 4 Sample 2

k	Observed frequency f_k	Expected frequency $e_k = np_k$	Probability p_k
0	1	0.000	0.000977
2	2	0.016	0.001953
3	9	0.087	0.008789
4	17	0.346	0.016602
5	14	1.099	0.013672
6	20	2.906	0.019531
7	36	6.590	0.035156
8	39	13.076	0.038089
9	51	23.062	0.049805
10	41	36.607	0.040039
11	62	52.824	0.060547
12	51	69.873	0.049805
13	74	85.315	0.072266
14	65	96.730	0.063477
15	65	102.360	0.063477
16	59	101.547	0.057617
17	60	94.816	0.058594
18	39	83.612	0.038089
19	43	69.851	0.041992
20	33	55.438	0.032227
21	37	41.903	0.036133
22	29	30.233	0.028320
23	24	20.865	0.023438
24	26	13.800	0.025391
25	20	8.762	0.019531
26	25	5.349	0.024414
27	19	3.145	0.018555
28	12	1.783	0.011719
29	9	0.976	0.008789
30	8	0.516	0.007812
31	10	0.264	0.009766
32	7	0.131	0.006836
33	5	0.063	0.004883
34	1	0.029	0.000977
35	1	0.013	0.000977
36	3	0.006	0.002930
37	3	0.003	0.002930
41	1	0.000	0.000977
42	2	0.000	0.001953
45	1	0.000	0.0009777
Total	$\Sigma f_k = 1024$	$\Sigma e_k = 1024.000$	$\Sigma p_k = 1.000000$

Table 5 Sample 2

k	Observed frequency f_k	Expected frequency $e_k = np_k$	Deviation $(f_k - e_k)^2/e_k$
≤ 7	99	11.047	700.225
8	39	13.076	51.394
9	51	23.062	33.844
10	41	36.607	0.527
11	62	52.824	1.594
12	51	69.873	5.098
13	74	85.315	1.501
14	65	96.730	10.408
15	65	102.360	13.636
16	59	101.547	17.827
17	60	94.816	12.784
18	39	83.612	23.803
19	43	69.851	10.322
20	33	55.438	9.081
21	37	41.903	0.574
22	29	30.233	0.050
23	24	20.865	0.471
24	26	13.800	10.787
25	20	8.762	14.415
≥ 26	107	12.280	730.624
Total	$\Sigma f_k = 1024$	$\Sigma e_k = 1024.000$	$\chi 2 = \Sigma (f_k - e_k)^2/e_k = 1648.963$

Table 6 Sample 2 Goodness of Fit Test Result

Item	Symbol	Value
Sample size	N	1024
Estimated λ	$\hat{\lambda}$	15.873
Degrees of freedom	ν	18 (k = 7, ..., 26)
Chi-square	χ^2	1648.96 (tail area = 0.00%)
Significance level	α	5.0%
Significance point	χ^2_α	28.8693
Hypothesis	H_0	Reject

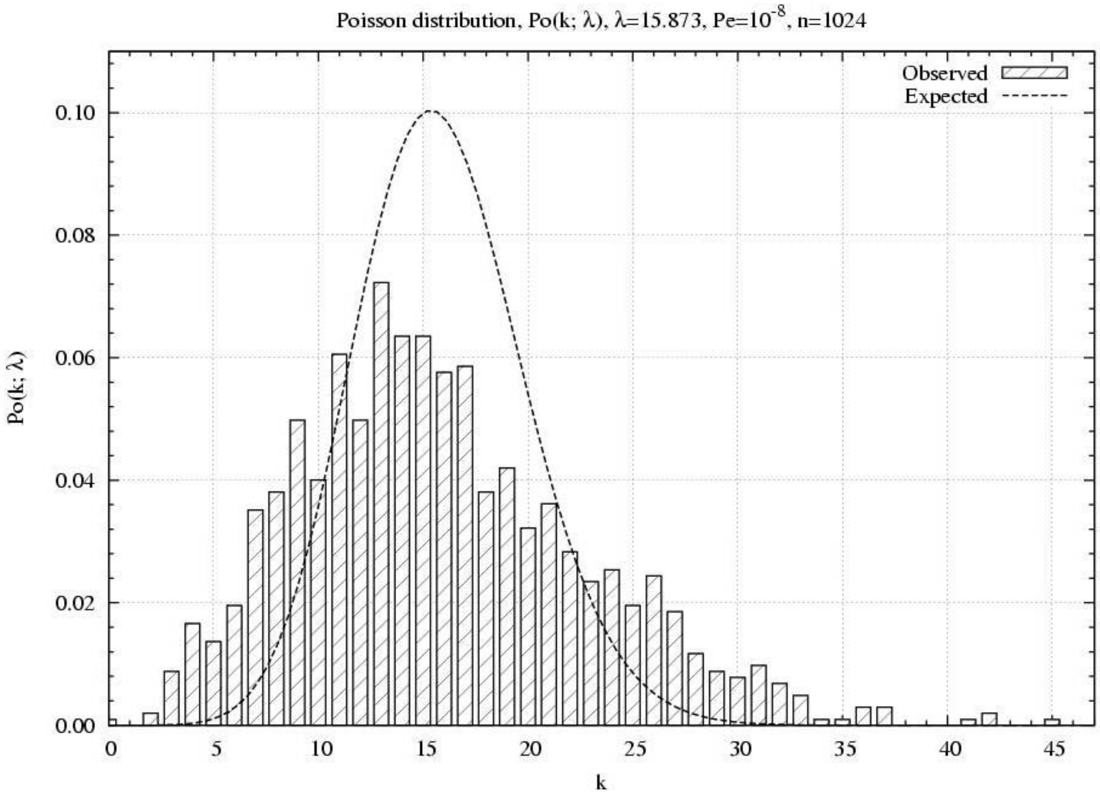


Figure 9 Sample 2

6. Conclusion

The method for testing a Poisson error generator describe in this White Paper, have been adopted in the March 2003 ITU-TO.182 recommendations for standards related to evaluation of Optical Transport Networks (OTNs).

Next-generation OTN technologies were standardized in ITU-T G.709 in February 2001 and the various manufacturers are increasingly bringing OTN-compliant equipment to market. The OTN standards provide an error correction function called Forward Error Correction (FEC) and precise evaluation of FEC performance requires measuring instruments supporting random error insertion. However, the randomness of inserted errors varies according to the manufacturer and cause problems such as dispersion in measurement results, and required standardizing a method for evaluating FEC performance. As a solution to this situation, we verified that it is possible to obtain accurate measurement results of FEC performance by inserting random errors fitting a Poisson distribution and proposed standardizing on this method. Naturally, both the Anritsu MP1590B Network Performance Tester and MP1595A 40G Analyzer have the random error insertion function for evaluating FEC performance built-in and this proposed method for testing a Poisson error generator will assure accurate measurement of FEC performance. Anritsu is licensing its patented Poisson error generator test method free of charge to ITU-T in the expectation of promoting future standardization and progress in worldwide telecommunications.

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