Differential noise figure: concepts and measurement approaches

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[Summary] While noise figure has been routinely measured for decades, the measurement of noise figure of a differential device is a somewhat newer problem and has often been treated very simply. There are cases when a simple treatment can result in significant errors and offering a measurement solution to handle more general cases may be very useful to the community. This paper will discuss principles behind the differential noise figure measurement and measurement approaches that can deal with various contingencies.

1 Introduction

Differential active devices have been increasingly popular in modern RF/microwave/millimeter-wave design for reasons of increased efficiency, better noise immunity, simplicity of design and other reasons. For system design, measuring the noise contribution of these devices is important but the measurement process has not been treated as extensively as for some other parameters. To begin, it may be useful to look at the history of noise figure definitions in general.

While it may not have been called 'noise figure' yet, the concept of propagating signal-to-noise ratios has been in use for nearly 100 years[1] (and perhaps longer). The term 'noise figure' was popularized more than 70 years ago by Friis[2] and others. The most currently used definition was codified by the IRE in 1963 to be the noise power at the output of the device under test (DUT) divided by the noise power at the output due to a matched termination at the input of the DUT at a temperature of 290K[3].

The intent of noise figure has always been that of a simple, single number to characterize device noise that is perhaps not complete[4]. As an example, this definition does not take into account changing source impedance and that description requires the more complete set of noise parameters[e.g., 5]. The noise figure definition above has presented challenges[e.g., 6] for some single-ended cases when multiple 'modes' or transmission paths are in play. These same issues affect the differential/multiport noise figure measurement but the differential case are also raises questions on how the inputs should be terminated (assuming something like a four-port amplifier) and how the noise from the input terminations should be related.

Before continuing, the concept of cross-correlation should be discussed and the word here is used in the statistical sense[7] (in the rest of this paper, the terms 'correlation' and 'cross-correlation' are used interchangeably). If one considers the noise voltage waveforms from two 50Ω terminations that are isolated from each other, these should be uncorrelated. If the temperatures of the terminations are the same, the root-mean-square amplitudes of the waveforms may be the same but the correlation (sum of products of complex amplitudes) will go to zero because the two signals arise from unsynchronized carrier motion in the two terminations. Conversely, if one had a single termination connected to an ideal, lossless splitter and observed the noise waveforms from the splitter outputs, these would be highly correlated.

Returning to the question (in multiport noise measurement) on how the noise of the input terminations should be related: should they be assumed to be correlated or uncorrelated? Uncorrelated is certainly easier from a measurement perspective (separate terminations will automatically provide this) and it works out the noise from ports of an entirely passive network must be uncorrelated[8] so any fixturing between terminations and DUT will not change the lack of correlation. Further, using the concept that noise figure should be a 'simple' number, it makes more sense to define noise figure to have uncorrelated inputs and leave the issues of correlated inputs to a more expansive noise parameter definition[5]. Fortunately, as several authors have pointed out[e.g., 4], many of the other potential multiport noise measurement questions also resolve themselves by enforcing a need for self-consistency. As an example, the gain definition (used to compute the noise power at the output due to those input terminations) must include differential gain and mode conversion gain from common-mode to differential. Thus, a working definition of differential noise figure is the noise power contained in the differential mode at the output divided by that differential noise power due to uncorrelated input terminations at 290K. This causes the differential noise figure to vary roughly inversely with DUT differential gain assuming constant added noise power.

With a definition in place, the next topic is how should single-ended noise figure be measured? The use of a Vector Network Analyzer will be assumed since it can acquire all S-parameters (the DUT gain is needed to compute noise figure_ and make match corrections along the way[9] but other receivers can be used. Even then, there are several basic methods of measuring noise power/noise figure but, for simplicity, the 'cold source' method[e.g., 9] will be used where noise power from the DUT (with inputs terminated) is measured directly with some suitable receiver calibration. In a single-ended case, the VNA generally computes noise power by summing a collection of samples (akin to noise voltages) from the DUT output, in an average mean-square fashion, that is sometimes expressed as

$$noise\ power = \overline{|b_i|^2} = \sum b_i b_i^* / N \quad (1)$$

Where b_i is the wave (with units proportional to voltage) received at the i^{th} port of the instrument, * denotes a complex conjugate and N is the number of measurements taken. Typically N is large (many thousands commonly) to reduce the amount of data variation but not so large that DUT drift starts to have an effect. This is a simplified expression ignoring receiver and residual noise calibrations. What has to change in the differential measurement?.

2 Measurement Approaches

In recent years, two approaches have generally been used to measure noise figures of differential devices: measure single-ended noise figure (averaging the results from the two output ports) which intrinsically assumes that the noise from the two ports is uncorrelated or use a balun on the output and measure single-ended noise figure (and de-embed the loss of the balun). Depending on the DUT (and on the balun), these can be perfectly valid results. There are, however, DUTs/baluns where problems can occur with these approaches.

If the noise-dominant structure of the device is like that in the top of Figure 1 (and the noise mechanisms are not arising from a common node), the uncorrelated assumption can be perfectly reasonable and there are many amplifier designs in existence where this is the case. If, however, the noise-dominant stage is more like a differential pair (bottom of Figure 1) or the output line coupling is quite strong, the uncorrelated assumption may be problematic.

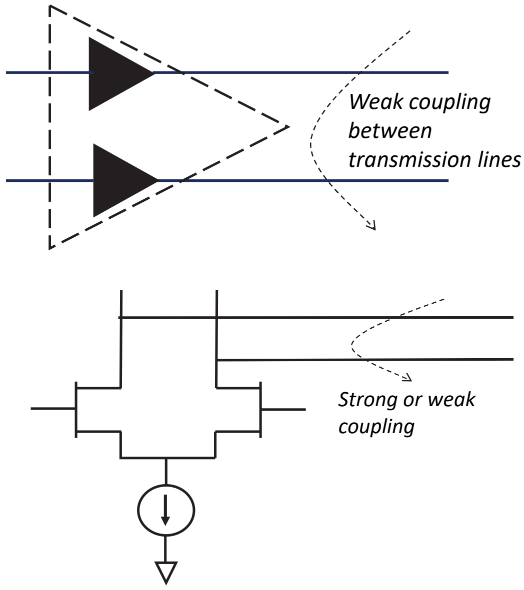

Figure 1 Two examples of amplifier topologies that can affect differential noise figure measurement assumptions. This distinction affects how noise figure can be measured accurately.

As discussed, the use of a DUT output balun is a simple, obvious, and frequently successful solution to performing the measurement (as would the use of an in-phase combiner should common-mode noise figure be needed). Many authors have pointed out[e.g., 10) to 13)] that there are a number of ways that this measurement can be processed and there are several error sources to consider. Even the most basic approach will de-embed the differential loss of the balun and perhaps consider its thermal noise addition to the measurement. Such a basic method, however, may not capture distortions in noise power correlation. Imbalances will lead to common-mode DUT noise power being coupled into the single-ended balun output and other incorrect allocations. At another level of analysis, the match interaction between the balun and the DUT could change the degree of noise power correlation.

An improved, corrected balun approach would be to compute an effective correlated power that is delivered to the common balun node taking into account the imbalances. If the single-ended DUT noise powers are separately known (these are added measurements of the DUT but require no added hardware), then the total differential power can be computed. A variety of permutations of techniques exist to do this correction[10) to 11)]. One such approach is to calculate the amount of common-mode DUT noise power that makes it to the balun output and what amount of differential mode DUT noise power does not.

It would be useful to have some understanding of the approximate size of errors introduced by not performing an improved balun correction. If the DUT outputs were uncorrelated, the error would be essentially zero[10)]. If the DUT outputs were highly correlated, then a simulation vs. balun imbalance can be performed to visualize the error. The results of such a simulation (for a nominal 4 dB balun insertion loss, 5 dB noise figure DUT with 20 dB of differential gain and moderate correlation) are shown in Figure 2. While levels of balun imbalance vary, 10 degrees of phase imbalance for a broadband balun is not unusual which could result in an added 0.5 dB of error. Even in terms of cable length matching, 10 degrees at 50 GHz (using common VNA test cables) arises from only a 125 µm length difference so such phase imbalances do practically occur. Note that the sensitivity to magnitude imbalance is somewhat less. The additional balun corrections have been implemented in the MS464X VectorStar VNAs to enable the increased accuracy when the user wishes to employ a balun.

Another approach may be to consider directly measuring the correlation signal and this has also been implemented in the MS464X VectorStar VNA differential noise figure option. Equation 2 shows the result upon expanding the mean-of-sum-of-squares of the differential signal (ports i and j are defined to be the DUT output ports). Again, receiver calibration terms are omitted for simplicity.

$$noise\ power\ diff = \frac{1}{2}\overline{|b_i - b_j|^2} = \frac{\Sigma(b_i - b_j)(b_i^* - b_j^*)}{2N} = \frac{\Sigma\left(|b_i|^2 + |b_j|^2 - 2Re(b_i b_j^*)\right)}{2N} \quad (2)$$

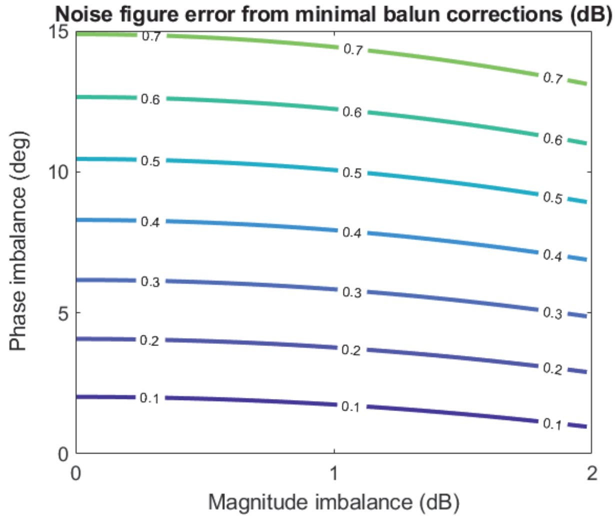

Figure 2 The error in noise figure resulting from balun imbalance can exceed 0.5 dB (for the basic balun method where only the differential loss of the balun is considered) for 10 degrees of phase imbalance.

The first two terms in the numerator form the average of the single-ended powers (the result from the uncorrelated method). The last term is related to the mean of the real part of the cross-correlation of the two waveforms. This is a physically reasonable result: if the waveforms are indeed uncorrelated, the last term will sum to zero and the whole equation reduces to the simple 'assume disjoint amplifiers' case discussed earlier.

Instead of using a balun and corrections to get at this quantity, it could be useful to measure it directly and this is possible if the b_i and b_j receivers in the VNA (or other instrument) are phase synchronized[14)]. Since these are complex quantities, a phase calibration is now required as well the traditional receiver and noise calibrations. This can be performed using a common sinusoidal source but the reference plane must be consistent relative to the DUT for both receiver paths.

A simplified diagram of the measurement setup is shown in Figure 3. The key differences from a conventional setup are the coherently clocked receivers (the downconverters share a local oscillator and the analog-to-digital converters (ADCs) share a clock) and the phase reference plane. Note that the two receiver chains need not be identical but they typically will have similar net gain and noise levels.

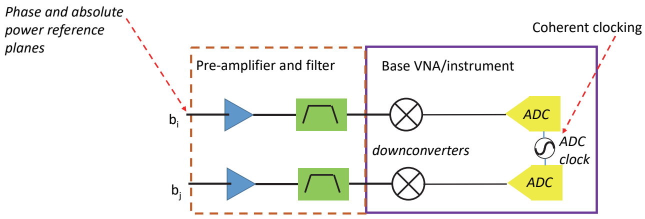

Figure 3 A diagram of the correlated noise figure measurement setup. The instrument must have coherent receivers and appropriate calibrations for this method. The pre-amplifier and filter are used to reduce receiver noise contributions and may be internal or external.

Since only one measurement is necessary and no balun characterization is needed, this approach is somewhat simpler than the balun-based methods. Also, differential and common-mode noise figures are available simultaneously.

3 Measurement Comparisons

As a first comparison, the noise power from a differential amplifier (and the output stage is purely differential so substantial correlation is expected) covering low frequencies to 4 GHz was measured using both the classical assume-uncorrelated approach and using the new direct correlation approach. The noise powers are plotted in Figure 4. The assume-uncorrelated approach will generate equal differential and common-mode noise powers and this is indicated in the figure. Using the direct correlated approach, these powers are clearly no longer equal and this was expected. The common mode noise power is quite low at some frequencies and the degree of correlation may not be constant. The separation from the uncorrelated assumption appears more unequal than one might expect based on the previous section but this is at least partially due to the data being plotted on a log scale (and this also emphasizes the variation in the common mode power). Because the common mode gain for this DUT is to 40 dB below that of the differential mode, the common-mode noise figure is much higher than that for the differential mode. This result is intuitive since an injected common mode signal at the input will be heavily attenuated at the output while the DUT does add some noise so the signal-to-noise ratio for the common mode will be significantly degraded.

Another general observation is that the assume-uncorrelated method will tend to understate noise power and noise figure for the dominant mode (differential in this case) and will tend to overstate noise power and noise figure for the non-dominant mode.

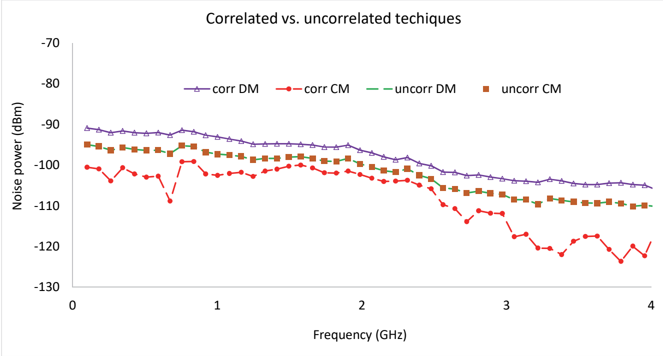

Figure 4 The assume-uncorrelated approach cannot distinguish common-mode and differential noise powers and forces them to be equal. For a highly correlated DUT, a treatment dealing with the correlation produces a rather different result. Noise power is plotted here in a 100 kHz bandwidth.

Another example measurement for a millimeter-wave differential amplifier is shown in Figure 5. The conclusions are similar to those of the previous example (plotted in terms of noise figure now) but illustrate that the methods and approaches translate well across frequency. The only execution differences are the receiver pre-amplifier/filter structure must be appropriate for the frequencies being tested.

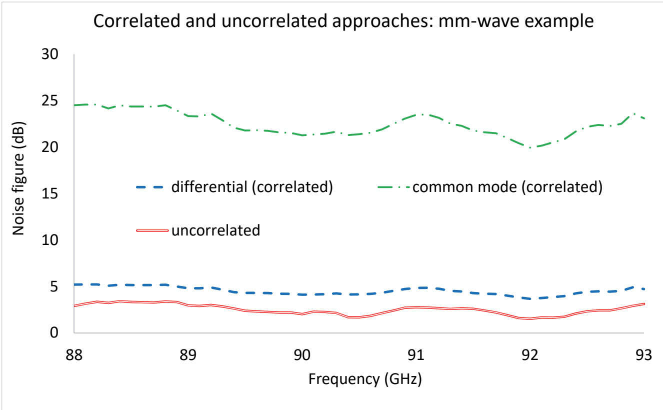

Figure 5 In this mm-wave amplifier example, the assume-uncorrelated approach underestimates the noise figure (more of the noise power was in the dominant mode than the uncorrelated assumption would suggest). The common-mode noise figure is quite high since the common-mode gain of the DUT is very low (not unusual for differential amplifiers).

As the balun-based approaches have been commonly used recently, a comparison between these methods and the direct correlated method would also be useful. As discussed in the previous section, there are several levels of correction that can be employed when using a balun. The simplest (termed 'basic' in Figure 6) only takes into account the differential-to-single-ended insertion loss of the balun and ignores imbalance, mismatch, etc. As seen in the figure, this can result is a significant amount of variation and error although, in this example, the electrical lengths are fairly long so some of these errors-by-neglect are accentuated. The 'corrected balun' approach takes into account the full S-parameters of the balun and reduced the variation to be somewhat similar to that of the direct-correlated approach although there are still some larger excursions that likely result from residual mismatch effects.

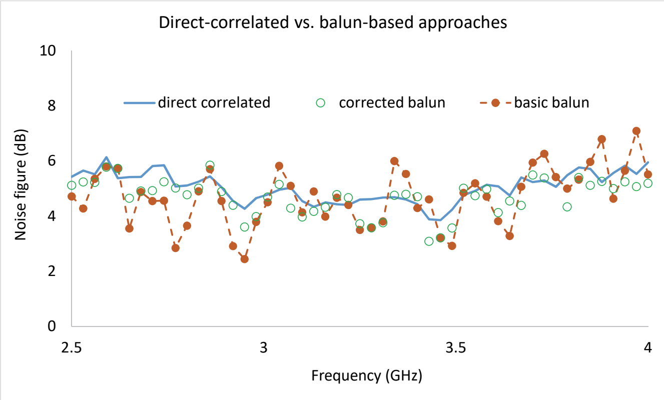

Figure 6 Among the class of balun-based approaches, there are several levels of correction. Significant variation and error can be experienced by only accounting for the insertion loss ('basic balun'). A 'corrected balun' approach and the direct-correlated approach produce more similar results.

4 Uncertainties and Factors Involved

Noise figure uncertainty is a complex topic[e.g., 15) to 17)] and many factors contribute. For single-ended, cold-source noise figure measurements, terms to consider include.

- Absolute power calibration uncertainty (including match effects)
- Receiver calibration uncertainty (including match effects)
- S-parameter uncertainty (for DUT gain)
- Data jitter due to limited acquisition length and system noise floor
- Receiver linearity
- Repeatability

In the differential case, all of these still apply but additional terms arise due to correlation handling. For an uncorrelated DUT, the only significant new terms are uncertainties in the phase calibration (for the direct correlation method) and balun characterization uncertainties (for the corrected balun method). The basic balun method will have uncertainty terms related to balun imbalance as discussed in an earlier section. Generally, these new terms do not dominate the overall uncertainty (power calibration and data jitter often do).

When the DUT has correlated noise outputs, the uncorrelated method will have an additional noise power uncertainty proportional to the cross-correlation term in Equation (2). For a DUT with completely correlated outputs, this term will approach the average of single-ended powers so a 3 dB uncertainty addition is possible. For the basic balun approach, the effect of neglected imbalance increases as the common-mode noise power from the DUT increases so the error from the simplification could easily exceed 0.5 dB for a typical broadband balun. The corrected balun and direct-correlation approaches will show increased uncertainty as well (since the correlation term is larger, uncertainties in its determination will play a larger role) but typically the added impacts are on the scale of tenths of a dB.

As a very simple example, one can calculate the added uncertainty as the relative size of the correlation term increases (coefficient of 0 for completely uncorrelated outputs and 1 for completely correlated) assuming the worst-case phasing (which is not unusual for a well-balanced amplifier with a differential core). The direct-correlation uncertainty term arises mainly from residual imprecision in the phase reference planes. The results, plotted in Figure 7, show the increasing differences at higher levels of correlation.

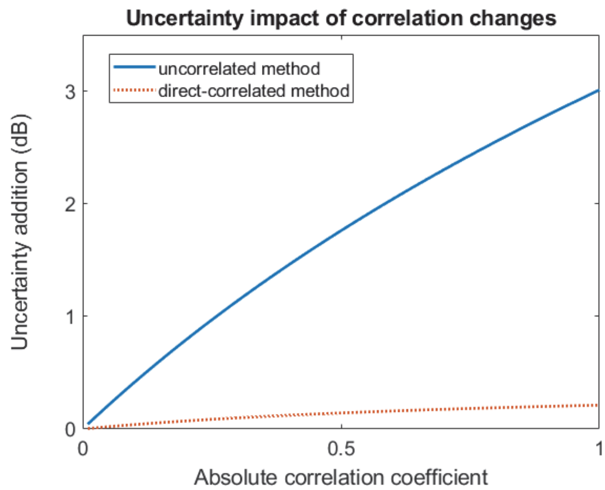

Figure 7 As discussed in the text, the added noise figure error in assuming an uncorrelated structure can approach 3 dB when the DUT is actually highly correlated.

5 Conclusions

Differential noise figure measurements are increasingly important and some existing methods have been prone to large errors for certain classes of DUTs. The DUTs of particular concern are those where the noise waveforms between the output ports are partially or highly correlated. In these cases, a direct-correlation approach offers improvements over a classical approach that assumes the output waveforms are uncorrelated and over a measurement using a balun where only the insertion loss of the balun is accounted for. The direct correlated approach produces similar t results to those of a more advanced balun-based approach (that corrects for imbalance) but the measurement is easier to perform.

References

1) F. B. Llewellyn, "A study of noise in vacuum tubes and attached circuits", *Proc I.R.E.*, vol. 18, pp. 243-266, February 1930.
2) H.T. Friis, "Noise figures of radio receivers," *Proc. IRE*, vol. 32, pp. 419-422, July 1944.
3) "Description of the Noise Performance of Amplifiers and Receiving Systems," Sponsored by IRE subcommittee 7.9 on Noise, *Proc. of the IRE*, pp. 436-442, March 1963.
4) J. Randa, "Noise characterization of multiport amplifiers," *IEEE Trans. Micr. Theory Techn.*, vol. 49, pp. 1757-1763, Oct. 2001.
5) L. Boglione, "Generalized determination of device noise parameters," *IEEE Trans. Micr. Theory Techn.*, vol. 65, pp. 4014-4025, Oct. 2017.
6) S. A. Maas, Microwave Mixers, 2nd ed, Artech House, 1993.
7) J. Keilson, N. Mermin and P. Bello, "A theorem on cross correlation between noisy channels," *IRE Trans. On Info. Theory*, vol. 5, pp. 77-79, Jan. 1959.
8) S. Wedge and D. Rutledge, "Noise waves and passive linear multiports," *IEEE Micr. Guid. Wave Lett.*, vol. 1, pp. 117-119, May 1991.
9) N. Otegi, J. M. Callantes, and M. Sayed, "Cold-source measurements for noise figure calculation in spectrum analyzers," *67th ARFTG Conf. Dig.*, June 2006, pp. 223-228.
10) Y. Andee, C. Arnaud, F. Graux, and F. Danneville, "De-embedding differential noise figure using the correlation of noise output waves," *84th ARFTG Conf. Dig.*, Dec. 2014, pp. 1-4.
11) M. Robens, R. Wunderlich and S. Heinen, "Differential noise figure de-embedding: a comparison of available approaches," *IEEE Trans. Micr. Theory Techn.*, vol. 59, pp. 1397-1407, May 2011.
12) Y. Chang, S. Lin, H. Liao, H. Chiou2, and Y. Juang, "On-wafer differential noise figure and large signal measurements of low-noise amplifier," *Proc. 39th Eur. Micr. Conf.*, Oct. 2009, pp. 699-702.
13) J. Dunsmore and S. Wood, "Vector corrected noise figure and noise parameter measurements of differential amplifiers," *Proc. 39th Eur. Micr. Conf.*, Oct. 2009, pp. 707-710.
14) J. Martens, "Measuring differential noise figure," *Microwave Journal*, vol. 61, Sept. 2018.
15) N. Otegi, J. M. Collantes and M. Sayed, "Uncertainty estimation in noise figure measurements at microwave frequencies," *Proc. 2005 IEEE Int. Wkshp. On Adv. Methods for Unc. Estim.*, Aug. 2005, pp. 84-89.
16) J. Martens, "On improving mm-wave noise figure measurements," *80th ARFTG Conf. Dig*, Dec. 2012, pp. 1-5.
17) M. Seelmann-Eggebert, B. Baldischweiler, B. Aja, D. Bruch and H. Massler, "Figures of uncertainty for noise measurements," *81st ARFTG Conf. Dig.*, June 2013, pp. 1-4.

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